EDGEWORTH BOX

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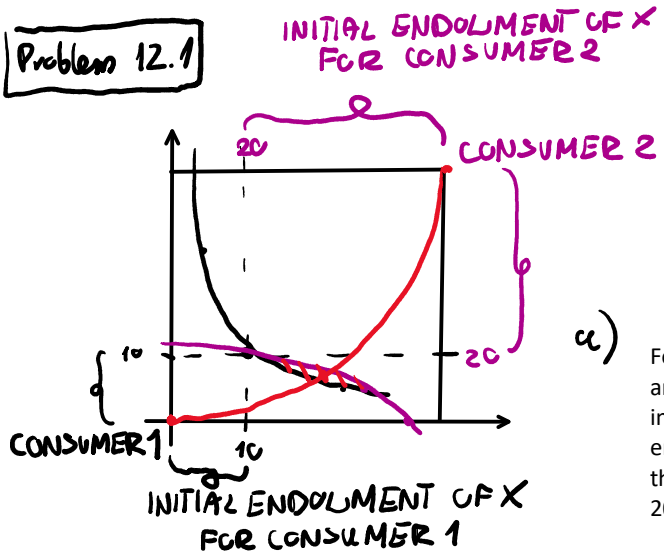
Problem 12.1



$U_1(x,y) = xy$

$U_2(x,y) = x^{1/2} y^{3/2}$

a)

For Consumer 1 and his initial endowment (10 of X and 10 of Y) his utility is equal to 100. To draw his indifference curve going through the initial endowment we need to find other bundles of X and Y that give the utility of 100. For example, X = 5 and Y = 20, or X = 20 and Y = 5. Depicted by black curve.

For Consumer 2 and his initial endowment (20 of X and 20 of Y) his utility is equal to 400. To draw his indifference curve going through the initial endowment we need to find other bundles of X and Y that give the utility of 400. For example, X = 6 and Y = 30, or X = 10 and Y = 25 (as an approximation). Depicted by purple curve.

b)

The red area between the curves represents the potential improvement for both consumers

c)

To find the Contract Curve the following set of equations needs to be solved:

$$\begin{cases} MRS_1 = MRS_2 \\ X_1 + X_2 = 30 \\ Y_1 + Y_2 = 30 \end{cases}$$

$MRS_1 = \frac{Y_1}{X_1}$ AND $MRS_2 = \frac{1}{3}\frac{Y_2}{X_2}$

SO WE HAVE:

$$\begin{cases} \frac{Y_1}{X_1} = \frac{1}{3}\frac{Y_2}{X_2} \\ X_2 = 30 - X_1 \\ Y_2 = 30 - Y_1 \end{cases} \Rightarrow \frac{Y_1}{X_1} = \frac{1}{3}\frac{30 - Y_1}{30 - X_1}$$

AFTER SIMPLIFICATION:

$$Y_1 = \frac{15X_1}{45 - X_1}$$

THIS IS CONTACT CURVE

DEPICTED IN RED IN THE FIGURE

d) IN EQUILIBRIUM:

$MRS_1 = \frac{P_x}{P_y}$ AND $MRS_2 = \frac{P_x}{P_y}$

FOR CONSUMER 1 WE SOLVE:

$$\begin{cases} \frac{x_1}{y_1} = \frac{P_x}{P_y} \\ P_x x_1 + P_x x_2 = 10P_x + 10P_y \end{cases}$$

THIS IS BUDGET EQUATION

THIS IS THE VALUE OF THE INITIAL ENDOWMENT

BY SOLVING THIS SET OF EQUATIONS WE GET:

$$\begin{cases} y_1 = 5\left(\frac{P_x}{P_y} + 1\right) \\ y_2 = 5\left(\frac{P_y}{P_x} + 1\right) \end{cases}$$

⇐ THIS IS DEMAND OF CONSUMER 1

ANALOGOSLY FOR CONSUMER 2, WE HAVE:

$$\begin{cases} \frac{1}{3}\frac{y_2}{x_2} = \frac{P_x}{P_y} \\ P_x x_2 + P_y y_2 = 20P_x + 20P_y \end{cases} \Rightarrow \begin{cases} y_2 = 15\left(\frac{P_x}{P_y} + 1\right) \\ x_2 = 5\left(\frac{P_y}{P_x} + 1\right) \end{cases}$$

TO FIND THE PRICE RATIO WE USE:

$$y_1 + y_2 = 30 \Rightarrow 5\left(\frac{P_x}{P_y} + 1\right) + 15\left(\frac{P_x}{P_y} + 1\right) = 30 \Rightarrow \frac{P_x}{P_y} = \frac{1}{2}$$

GIVEN THAT WE HAVE:

$$\begin{cases} x_1^* = 15 \\ y_1^* = 7.5 \end{cases} \quad \text{AND} \quad \begin{cases} x_2^* = 15 \\ y_2^* = 22.5 \end{cases}$$