Problem set 14

sobota, 25 stycznia 2025 12:29

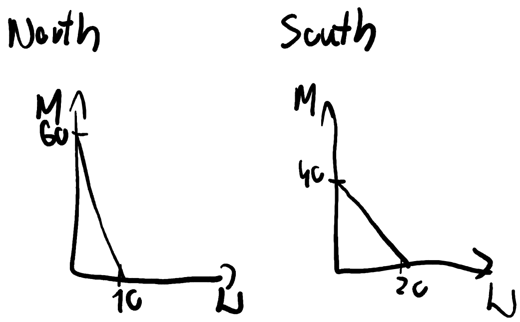
Problem 2

a) Production possibility frontiers:

North: $m = 60 - 6w$

South: $m = 40 - 2w$

Visualisation:



To calculate Marginal Rate of Transformation (MRT) it is convinient to transform PPFs to:

$$\begin{aligned} \text{North: } & 6w + M = 60 \\ \text{South: } & 2w + M = 40 \end{aligned} \Rightarrow \begin{cases} MRT_N = -\dfrac{\frac{\partial PPF}{\partial W}}{\frac{\partial PPF}{\partial M}} = \dfrac{-6}{1} = -6 \\ MRT_S = -\dfrac{\frac{\partial PPF}{\partial W}}{\frac{\partial PPF}{\partial M}} = -2 \end{cases}$$

The interpretation of MRT for North: to produce 1 more unit of wheat, they would have to decrease the production of milk by 6

The exchange rate is such that: $\dfrac{P_W}{P_M} = 4$

Therefore it holds that: $\underbrace{|MRT_S|}_{=2} < \underbrace{\dfrac{P_W}{P_M}}_{=4} < \underbrace{|MRT_N|}_{=6}$

Because of that, North will specialize in production of milk, and south in production of wheat. Intuitively, to produce 1 unit of wheat North would have to sacrifice 6 units of milk. Because of that, it is more benefitial for them to produce 6 milks and exchange them for 1.5 wheats.

On the other hand, the South needs to sacrifice only 2 milks for 1 wheat. If they produce milk, then, with the given exchange rate, they would only get 0.5 of wheat for 2 milks on the market. So it is better for them to produce wheat.

b) Viking comes in and proposes the exchange rate of: $\dfrac{P_W}{P_M} = 3$

Because the condition: $|MRT_S| < \dfrac{P_W}{P_M} < |MRT_N|$ Still holds for the new exchange rate, the production does not change.

c) $\dfrac{P_W}{P_M} \in (2, 6)$

Problem 3

a) PPF for region A:

$$\begin{cases} X_A = \sqrt{L_{AX}} \\ Y_A = \sqrt{\ldots} \end{cases} \Rightarrow \begin{cases} L_{AX} = X_A^2 \\ L_{AY} = Y_A^2 \end{cases}$$

we know that: $L_{AX} + L_{AY} = 100$

$$\begin{cases} X_A = \sqrt{L_{AX}} \\ Y_A = \sqrt{L_{AY}} \end{cases} \Rightarrow \begin{cases} L_{AX} = X_A^2 \\ L_{AY} = Y_A^2 \end{cases}$$

we know that: $L_{AX} + L_{AY} = 100$

so: $\boxed{X_A^2 + Y_A^2 = 100}$

This is PPF_A

For region B:

$$\begin{cases} X_B = \frac{1}{2}\sqrt{L_{BX}} \\ Y_B = \frac{1}{2}\sqrt{L_{BY}} \end{cases} \Rightarrow \begin{cases} L_{BX} = 4X_B^2 \\ L_{BY} = 4Y_B^2 \end{cases}$$

because: $L_{BX} + L_{BY} = 100$

we have: $\boxed{4X_B^2 + 4Y_B^2 = 100}$

PPF_B

b) $MRT = -\frac{MC_X}{MC_Y} = -\frac{P_x}{P_y} = MRS$

c)

If regions A and B produced only X they could produce 15 units of it. The same goes for Y. Therefore, joint PPF is:

$$X^2 + Y^2 = 15^2 \Rightarrow X^2 + Y^2 = 225$$

If $X = 12$, then $Y = 9$

d) $MRT_A = -\frac{2X_A}{2Y_A} = -\frac{X_A}{Y_A}$. PPF is $X_A^2 + Y_A^2 = 100$, so if $X_A = 8$, then $Y_A = 6$.

For these values: $MRT_A = -\frac{8}{6} = -4/3$

Interpretation: To produce one more unit of X region A would have to decrease the production of Y by 4/3.

Problem 4

a)

PPFs are given by:

$$\begin{aligned} 1&: C + M = 40 \\ 2&: C + 2M = 60 \end{aligned} \Rightarrow \begin{aligned} C &= 40 - M \\ C &= 60 - 2M \end{aligned}$$

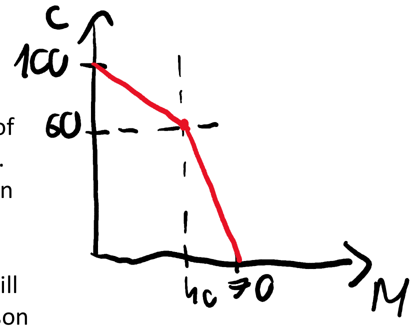
The aggregate PPF is given by:

$$\begin{cases} C = 100 - M & \text{if } M \in [0, 40) \\ C = 140 - 2M & \text{if } M \in [40, 70] \end{cases}$$

Graphically:

$$\begin{cases} C = 100 - M & \text{if } M \in [0, 40) \\ C = 140 - 2M & \text{if } M \in [40, 70] \end{cases}$$

Graphically:



Why is it like that?
If both individuals produce only C, they can get 100 units of it. So this is the value that the PPF needs to take for M = 0. Because the first individual has a comparative advantage in M (he needs to sacrifice only 1 unit of C for 1 unit of M, whereas the other person needs to sacrifice 2 units of C), then if we want to produce only a little amount of M he will be the one doing it. Specifically, for M <= 40 only the person 1 will be producing it, and so the slope will be -1. This gives us the first part of PPF.

For the second part, the second individual will start producing M, which will give us the slope of -2. The 140 arises because for M=40, C must be equal to 60, so that both lines would connect at this kink point.

b) The condition that needs to hold is: $|MRS| = |MRT|$

For $U = CM^2$ we have $MRS = 2\frac{C}{M}$

Because MRT is a piecewise linear function we need to consider two cases:

① $M \in [0, 40)$

Then: $|MRT| = 1$ ← slope of the line

We need to solve:

$$\begin{cases} 2\frac{C}{M} = 1 \\ C = 100 - M \end{cases} \Rightarrow \begin{matrix} M = 66\frac{1}{3} \\ C = 33\frac{1}{3} \end{matrix} \;\lightning$$

M is too large, so this cannot be our solution.

② $M \in [40, 70]$

$$\begin{cases} 2\frac{C}{M} = 2 \\ C = 140 - 2M \end{cases} \Rightarrow \begin{cases} M = 46\frac{2}{3} \\ C = 33\frac{1}{3} \end{cases}$$

← This is ok, so this is the optimum

c) $U(C, M) = CM$, so $|MRS| = \frac{C}{M}$

Again, two cases:

① $M \in [0, 40)$

$$\begin{cases} \frac{C}{M} = 1 \\ C = 100 - M \end{cases} \Rightarrow C = M = 50 \;\lightning$$

② $M \in [40, 70)$

$$\begin{cases} \frac{C}{M} = 2 \\ C = 140 - 2M \end{cases} \Rightarrow \begin{cases} M = 35 \\ C = 70 \end{cases} \;\lightning$$

$$\begin{cases} \tilde{M} = 1 \\ C = 100 - M \end{cases} \Rightarrow C = M = 50 \qquad \begin{cases} M = 2 \\ C = 140 - 2M \end{cases} \Rightarrow \begin{cases} \dots \\ C = 70 \end{cases}$$

If we find a contradiction in both cases, that means that we have a corner solution. As for Cobb-Douglas utility function the corner solutions with 0 production give utility = 0, the only viable solution is at the kink point, namely: C = 60 and M = 40. This will be the optimal consumption level.