Fundamental Theorems of Welfare Economics and Walras' Law

Market equilibrium

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Competitive equilibrium

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Walrasian equilibrium

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A set of prices such that each consumer is choosing her most-preferred affordable bundle, and demand equals supply in every market

Leon Walras (1834-1910) – French economist; early investigator of the general equilibrium theory

Equilibrium
in a perfectly competitive market

- For
  - a perfectly competitive market
  - with two consumers (A and B)
  - with strictly convex indifference curves describing preferences towards two goods (X and Y),

  the equilibrium in the Edgeworth box is in the tangency point of the indifference curves of the consumers.

- The slope of the tangent line in the equilibrium is equal to the negative of the equilibrium price ratio ($-p_X/p_Y$).

- $MRS_A = MRS_B = -p_X/p_Y$

General equilibrium

The total amounts of goods that consumers want to consume may differ from the available amounts of the goods.

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The market is in disequilibrium.

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Market prices need to adjust.

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A change in one market causes a change in another market. This will be continued until the general equilibrium (i.e., the equilibrium in each market) is reached.

First Theorem of Welfare Economics

When

- consumer preferences are well-behaved (locally non-satiated) and
- there exists a competitive equilibrium,

then this (competitive) equilibrium is a Pareto efficient allocation.

Local non-satiation of preferences – for any original bundle of goods, there is another bundle of goods arbitrarily close to the original bundle, but that is preferred to the original one.

There are some implicit assumptions in this framework: e.g., all consumers care only about their own consumption, no externalities, no transaction costs, perfect information.

First Theorem of Welfare Economics
Efficiency versus Fairness

- It guarantees that a competitive market will exhaust all of the gains from trade: An equilibrium allocation achieved by a set of competitive markets will necessarily be Pareto efficient.
- However, such an allocation may not have other desirable properties.
- In particular, the theorem says nothing about the distribution of economic benefits.
- The market equilibrium might not be a “fair” allocation:
  For example, if consumer A owns everything at the beginning, then she will own everything after trade.
  That will be efficient, but likely it will not be fair.

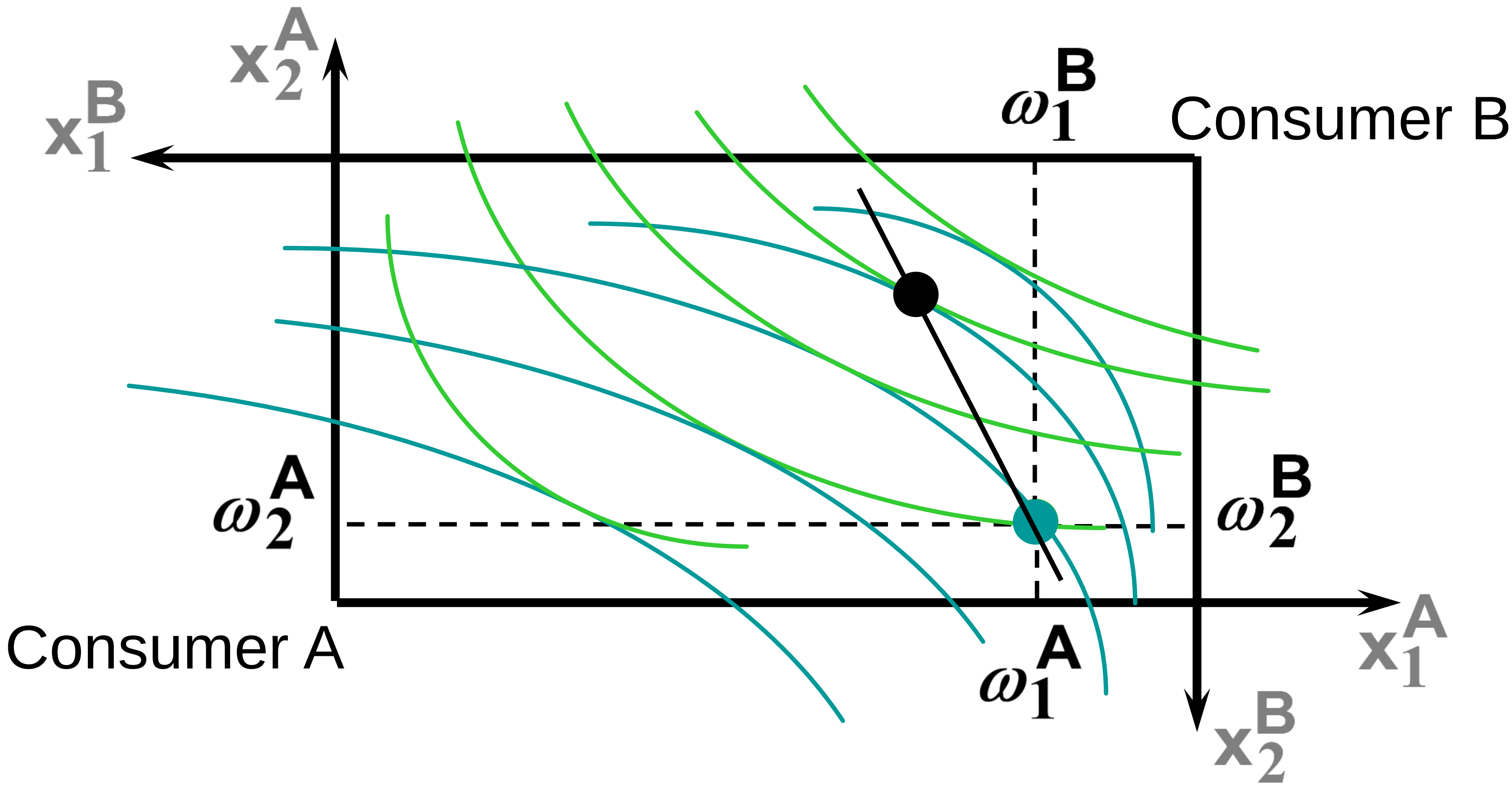
Second Theorem of Welfare Economics

- A “reversal” of the First Welfare Theorem
- It says that under some conditions, any Pareto efficient allocation is (part of) a competitive equilibrium.

- When preferences are convex, there is always a set of prices such that each Pareto efficient allocation is a competitive equilibrium for some initial endowments of goods.

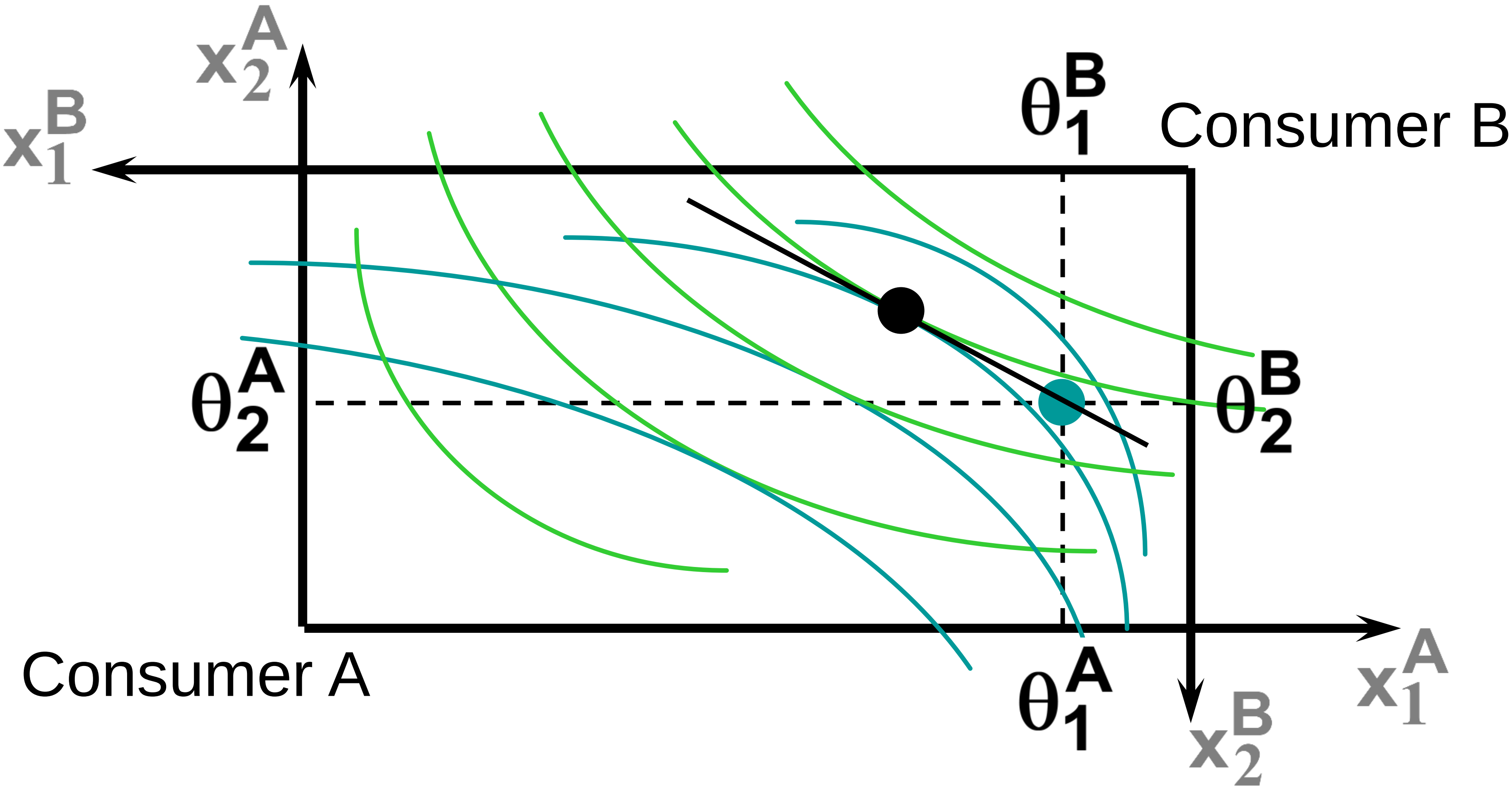
- Each Pareto efficient allocation means each point from the contract curve.
- Again, there are some implicit assumptions in this framework: e.g., all consumers are utility-maximizers, no externalities, no transaction costs, perfect information.

Second Theorem of Welfare Economics



Can the Pareto efficient allocation in black be reached by competitive trading, starting from the initial endowment ω? No.

Second Theorem of Welfare Economics



But the allocation in black can be reached by competitive trading, starting from the initial endowment θ.

In brief

- What is the relationship between competitive equilibrium and Pareto efficiency?
- First Welfare Theorem is about:
  Is any competitive equilibrium Pareto efficient?
- Second Welfare Theorem is about:
  Is any Pareto efficient allocation (part of) a competitive equilibrium?

Walras' Law

- *Net demand* or *excess demand* – a difference between how much of some good a consumer wants to consume and how much of that good she initially has.

- The value of aggregate excess demand is zero for any positive prices.

- It is zero for all possible sets of prices, not just for the equilibrium prices.
- The value of *aggregate excess demand* – the values of excess demands summed over all markets in the economy.

Walras' Law – Derivation

- Assumptions:
  - Two goods (1 and 2); Two consumers (A and B)
  - Every consumer has well-behaved preferences
  - For any positive prices (p_1, p_2), each consumer spends all of her budget

- Budget constraint of Consumer A

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

- Budget constraint of Consumer B

$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

x – gross demand
ω – initial endowment
(x – ω) – net/excess demand

Walras' Law – Derivation

The budget constraints:

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

We can sum them up

$$p_1(x_1^{*A} + x_1^{*B}) + p_2(x_2^{*A} + x_2^{*B})$$

$$= p_1(\omega_1^A + \omega_1^B) + p_2(\omega_2^B + \omega_2^B).$$

And rearrange

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0.$$

Walras' Law – Derivation

$$\mathbf{p}_1(\mathbf{x}_1^{*\mathbf{A}} + \mathbf{x}_1^{*\mathbf{B}} - \omega_1^{\mathbf{A}} - \omega_1^{\mathbf{B}}) +$$
$$\mathbf{p}_2(\mathbf{x}_2^{*\mathbf{A}} + \mathbf{x}_2^{*\mathbf{B}} - \omega_2^{\mathbf{A}} - \omega_2^{\mathbf{B}}) = \mathbf{0}.$$

$(x - \omega)$ is net demand/excess demand.

Walras' Law: The value of aggregate excess demand is zero for any positive prices.

Implication 1 of Walras' Law

If the value of aggregate excess demand in k-1 markets is zero, then the value of excess demand in the remaining k-th market is also zero.

Suppose there is no excess demand for good 1:

$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B = 0.$$

Then, Walras' Law

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$
$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

implies that for positive prices

$$x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B = 0.$$

Implication 2 of Walras' Law

For an economy with two goods, excess supply in the market of one good (i.e., negative excess demand) implies positive excess demand in the market of the other good.

Suppose there is excess supply of goods 1:

$$x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B < 0.$$

Then, Walras' Law

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) +$$

$$p_2(x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B) = 0$$

implies that

$$x_2^{*A} + x_2^{*B} - \omega_2^A - \omega_2^B > 0.$$

Implication 3 of Walras' Law

To determine the equilibrium for an economy consisting of k markets for different goods, we only need to find a set of equilibrium prices for $k - 1$ of these markets.

- Walras' Law implies that the market for good k will automatically have demand equal to supply if all other markets in the economy are in the equilibrium.
- This means there are really only $k – 1$ *independent* prices.
- We can choose one of the k prices and set it equal to a constant.
- It is often convenient to set one of the prices equal to 1 so that all other prices can be interpreted as being measured relative to it. Such a price is called a *numeraire* price.

Market equilibrium and planning

- It is difficult to reach the general equilibrium if all markets are not perfectly competitive.
- An efficient allocation may be achieved by central planning.
- However, market solutions are preferred (to central planning) because consumers and producers are able to better specify their preferences and production possibilities.