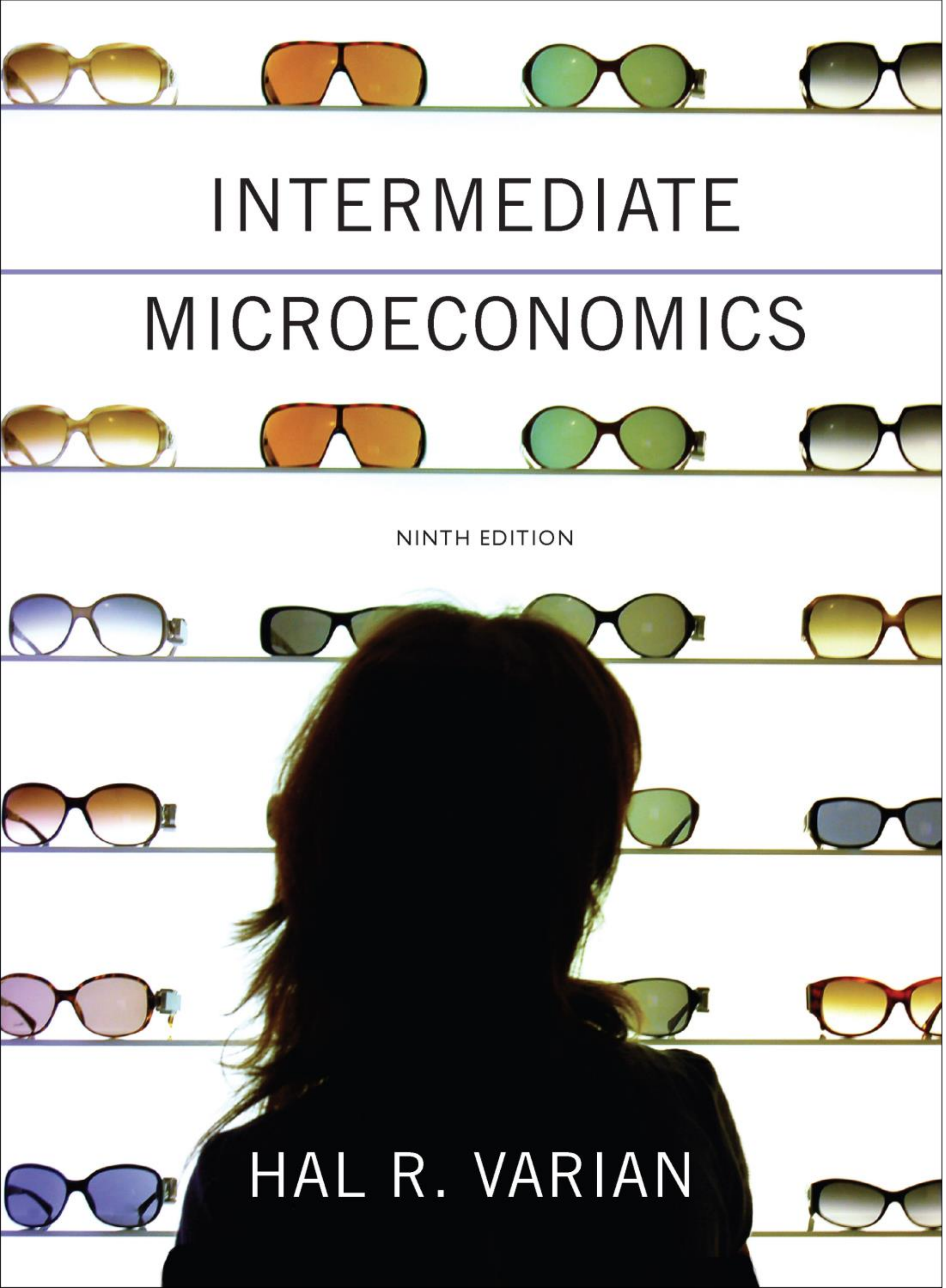



# Welfare

The presentation is based on slides by Hal R. Varian, *Intermediate Microeconomics*.

# Social Choice

- Usually, many Pareto efficient allocations exist.
- Different economic states will be preferred by different individuals.
- How can individual preferences be “aggregated” into a social preference over all possible economic states?

# Rational Preference Relation

- A preference relation is called rational if the preference order is both transitive and complete.

- Complete: Any two different bundles can be compared. $\forall(x,y) \in \mathbf{X}$, either $x \succeq y$, $y \succeq x$, or both.

- Transitive: A "consistency" requirement, enabling a ranking. If a consumer thinks that X is at least as good as Y and that Y is at least as good as Z, then the consumer thinks that X is at least as good as Z.
  For all choices *x*, *y*, and *z*, if $x \succeq y \wedge y \succeq z$ then $x \succeq z$

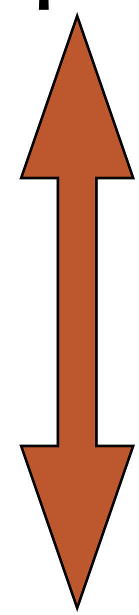
# Aggregating Preferences

- *x*, *y*, *z* denote different economic states.
- 3 agents: Bill, Bertha and Bob.
- Use simple majority voting to select a state?

# Aggregating Preferences

| Bill | Bertha | Bob |
|---|---|---|
| x | y | z |
| y | z | x |
| z | x | y |

**More preferred**



**Less preferred**

Bill, Bertha and Bob have rational preferences.

# Aggregating Preferences

| Bill | Bertha | Bob |
|---|---|---|
| x | y | z |
| y | z | x |
| z | x | y |

<u>Majority Vote Results</u>

x vs y: x beats y

y vs z: y beats z

x vs z: z beats x

# Aggregating Preferences

| Bill | Bertha | Bob |
|---|---|---|
| x | y | z |
| y | z | x |
| z | x | y |

<u>Majority Vote Results</u>

x vs y: x beats y
y vs z: y beats z
x vs z: z beats x

No socially
best alternative!

Majority voting does not always aggregate transitive individual preferences into a transitive social preference.

# Aggregating Preferences

| Bill | Bertha | Bob |
|---|---|---|
| x(1) | y(1) | z(1) |
| y(2) | z(2) | x(2) |
| z(3) | x(3) | y(3) |

Will ranking work?

# Aggregating Preferences

| Bill | Bertha | Bob |
|---|---|---|
| x(1) | y(1) | z(1) |
| y(2) | z(2) | x(2) |
| z(3) | x(3) | y(3) |

Will ranking work?

<u>Rank-Order Vote Results</u>
(lowest score wins)

x-score = 6
y-score = 6
z-score = 6

# Aggregating Preferences

| Bill | Bertha | Bob |
| --- | --- | --- |
| x(1) | y(1) | z(1) |
| y(2) | z(2) | x(2) |
| z(3) | x(3) | y(3) |

Will ranking work?

<u>Rank-Order Vote Results</u>
(lowest score wins)

x-score = 6
y-score = 6
z-score = 6

No state is selected!

Rank-order voting is indecisive in this case.

# Manipulating Preferences

- Most voting schemes are manipulable.
- The outcome of majority voting can depend on the order in which pairs of variants are voted. →
- The outcome of rank-order voting can be influenced by introducing new choice options.

# Manipulating Preferences

| Bill | Bertha | Bob |
|---|---|---|
| x | y | z |
| y | z | x |
| z | x | y |

Majority Vote Results

x vs y: x beats y (1)
y vs z: y beats z (2)
x vs z: z beats x (3)

For example:

- If the voting concerns only x and y, and y and z (1 and 2), then x will win.
- If the voting concerns only x and y, and x and z (1 and 3), then z will win.

# Manipulating Preferences

- Most voting schemes are manipulable.
- The outcome of majority voting can depend on the order in which pairs of variants are voted.
- The outcome of rank-order voting can be influenced by introducing new choice options.→

# Manipulating Preferences

| Bill | Bertha | Bob |
|---|---|---|
| x(1) | y(1) | z(1) |
| y(2) | z(2) | x(2) |
| z(3) | α(3) | y(3) |
| α(4) | x(4) | α(4) |

Rank-Order Vote

These are truthful preferences.

Bob introduces a new alternative (α).

# Manipulating Preferences

| Bill | Bertha | Bob |
| --- | --- | --- |
| x(1) | y(1) | z(1) |
| y(2) | z(2) | x(2) |
| z(3) | α(3) | y(3) |
| α(4) | x(4) | α(4) |

<u>Rank-Order Vote</u>

These are truthful preferences.

Bob introduces a new alternative (α).

And, then, he lies about his preference order.

# Manipulating Preferences

| Bill | Bertha | Bob |
|---|---|---|
| x(1) | y(1) | z(1) |
| y(2) | z(2) | **α(2)** |
| z(3) | α(3) | x(3) |
| α(4) | x(4) | y(4) |

Rank-Order Vote

Bob introduces a new alternative (α) and, then, lies about his preference order.

x-score = 8
y-score = 7
z-score = 6
α-score = 9

**z wins!**

# Manipulating Preferences

| Bill | Bertha | Bob |
| --- | --- | --- |
| x(1) | y(1) | z(1) |
| y(2) | z(2) | x(2) |
| z(3) | α(3) | y(3) |
| α(4) | x(4) | α(4) |

Rank-Order Vote

Bob introduces a new alternative (α) and, then, lies about his preference order.

**If he didn't lie:**

x-score = 7

**y-score = 6**

**z-score = 6**

α-score = 11

# Desirable Voting Rule Properties

1. If all individuals' preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.
2. If all individuals rank x before y, then so should the voting rule.
3. Social preference between x and y should depend on individuals' preferences between x and y only.

# Desirable Voting Rule Properties

- Kenneth Arrow's Impossibility Theorem: The only voting rule with all of properties 1, 2 and 3 is dictatorial.
- Dictatorship means a social outcome determined by a single individual.
- Implication is that a non-dictatorial voting rule requires giving up at least one of properties 1, 2 or 3.

# Social Welfare Function

1. If all individuals' preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.
2. If all individuals rank x before y, then so should the voting rule.
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Which one to give up in order to build a social welfare function?

# Social Welfare Function

1. If all individuals' preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.
2. If all individuals rank x before y, then so should the voting rule.
3. ~~Social preference between x and y should depend on individuals' preferences between x and y only.~~

**Which one to give up in order to build a social welfare function?**

# Social Welfare Function

1. If all individuals' preferences are complete, reflexive and transitive, then so should be the social preference created by the voting rule.
2. If all individuals rank x before y, then so should the voting rule.
3. Social preference between x and y should depend on individuals' preferences between x and y only.

There is a variety of voting procedures with both properties 1 and 2.

# Social Welfare Functions

- $u_i(x)$ is individual i's utility from <u>*overall*</u> allocation x.

- Utilitarian: $$W = \sum_{i=1}^{n} u_i(x).$$

- Weighted-sum: $$W = \sum_{i=1}^{n} a_i u_i(x) \text{ with each } a_i > 0.$$

- Minimax: (Rawlsian) $$W = \min\{u_1(x), \ldots, u_n(x)\}.$$

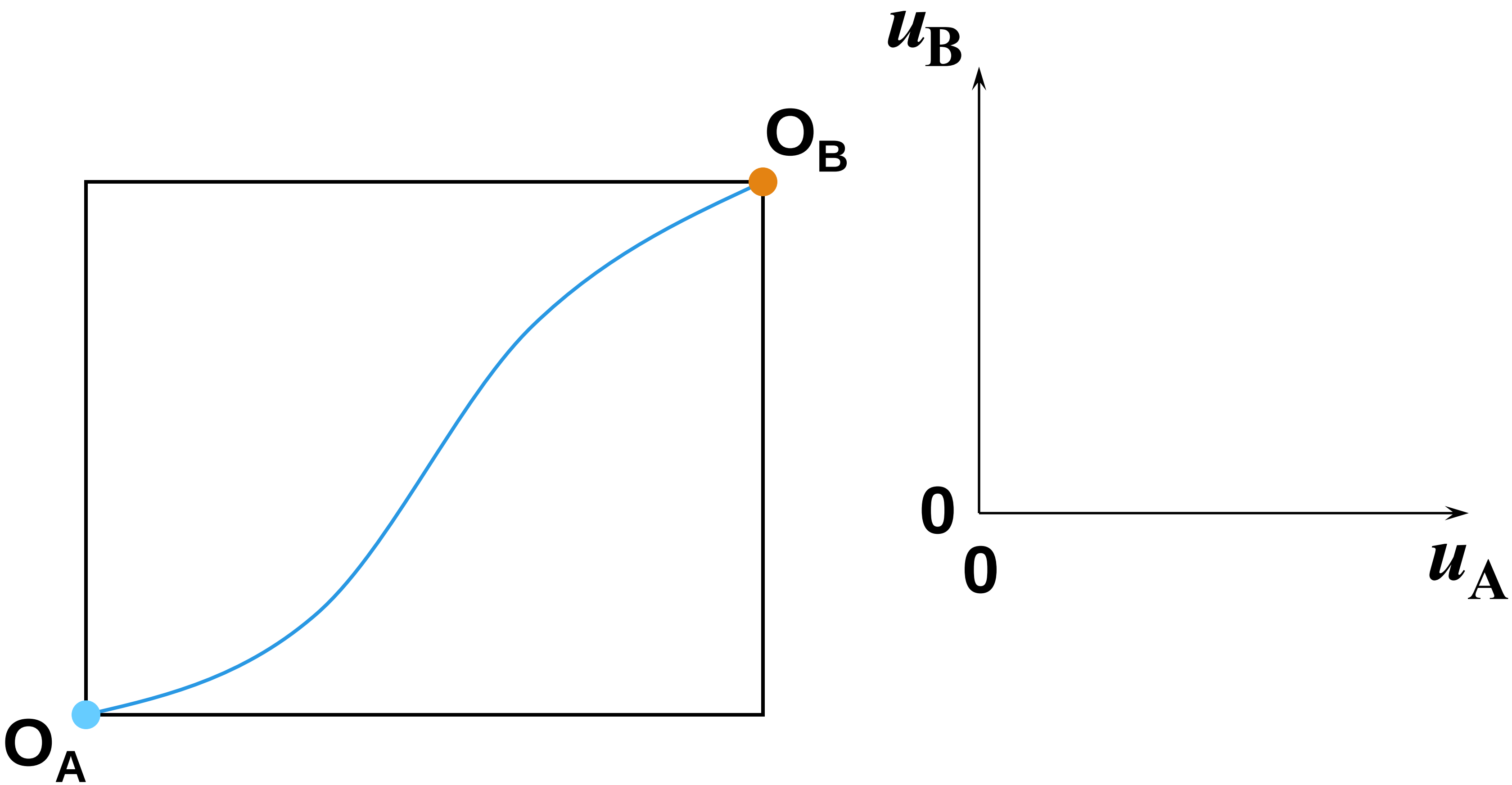
# Social Optima & Efficiency

- Any socially optimal allocation must be Pareto efficient.
- Why?

# Social Optima & Efficiency

- Any socially optimal allocation must be Pareto efficient.
- Why?
- If not, then somebody's utility can be increased without reducing anyone else's utility.
- That is, social suboptimality $\Rightarrow$ inefficiency.

# Utility Possibilities



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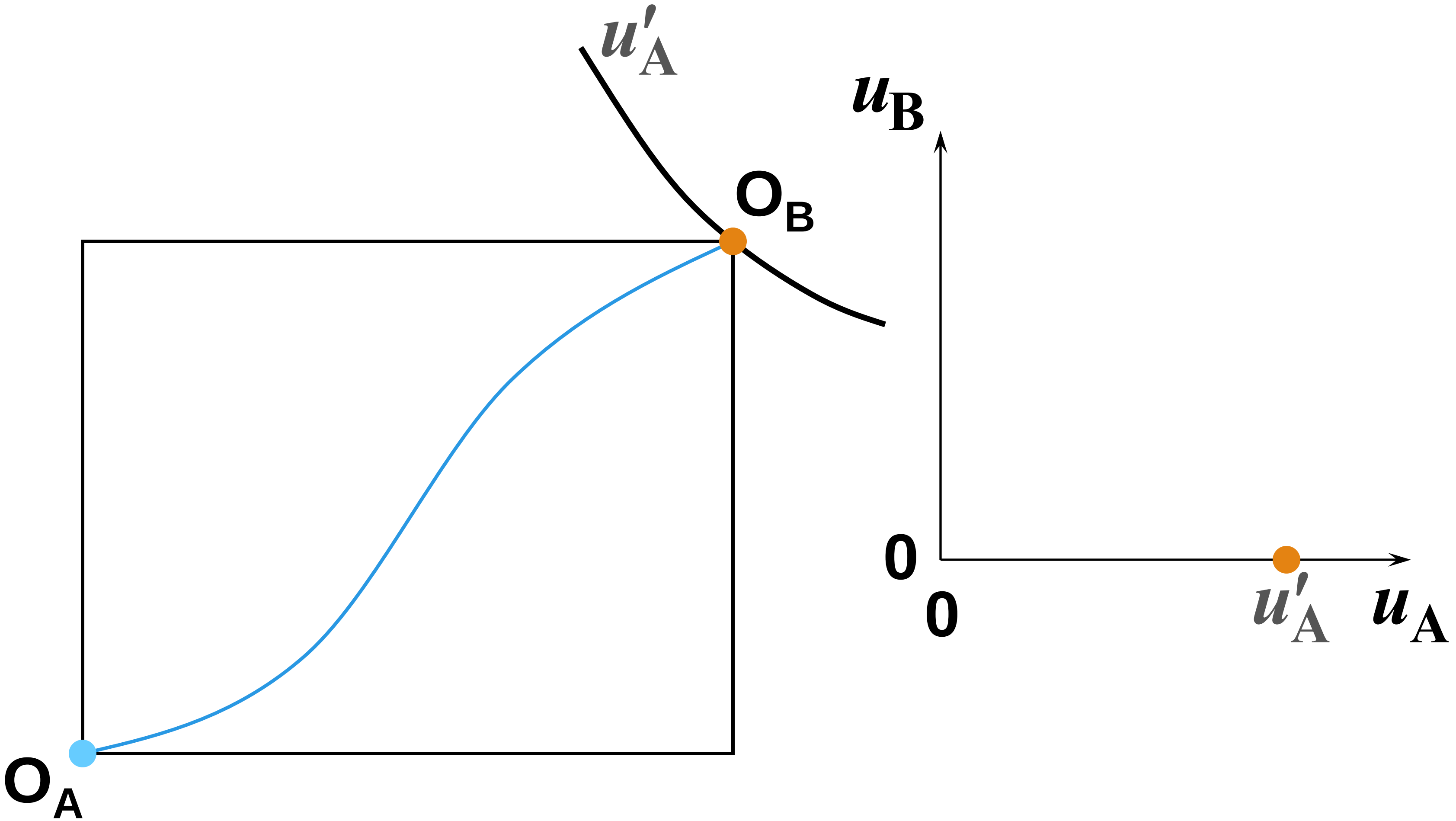

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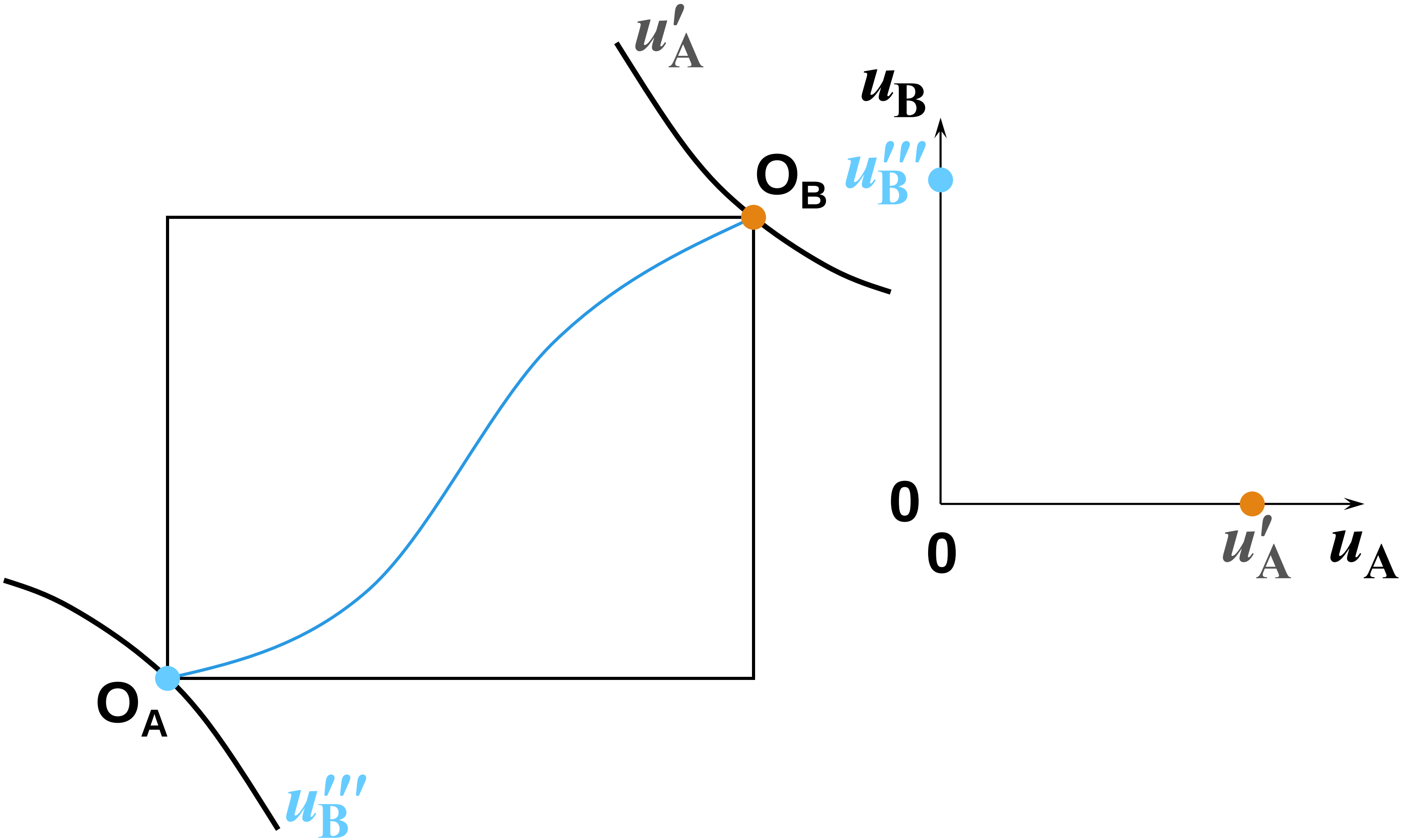

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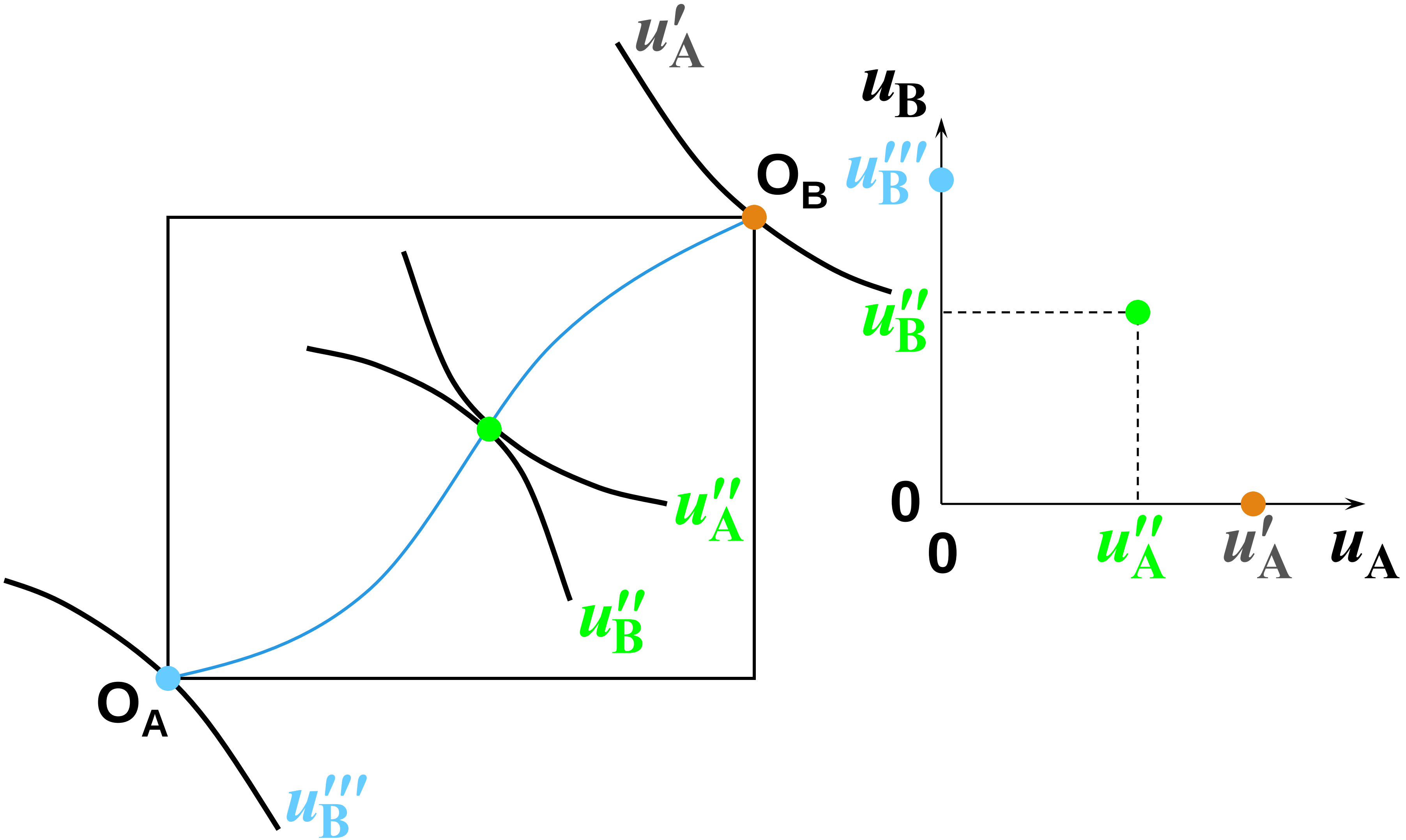

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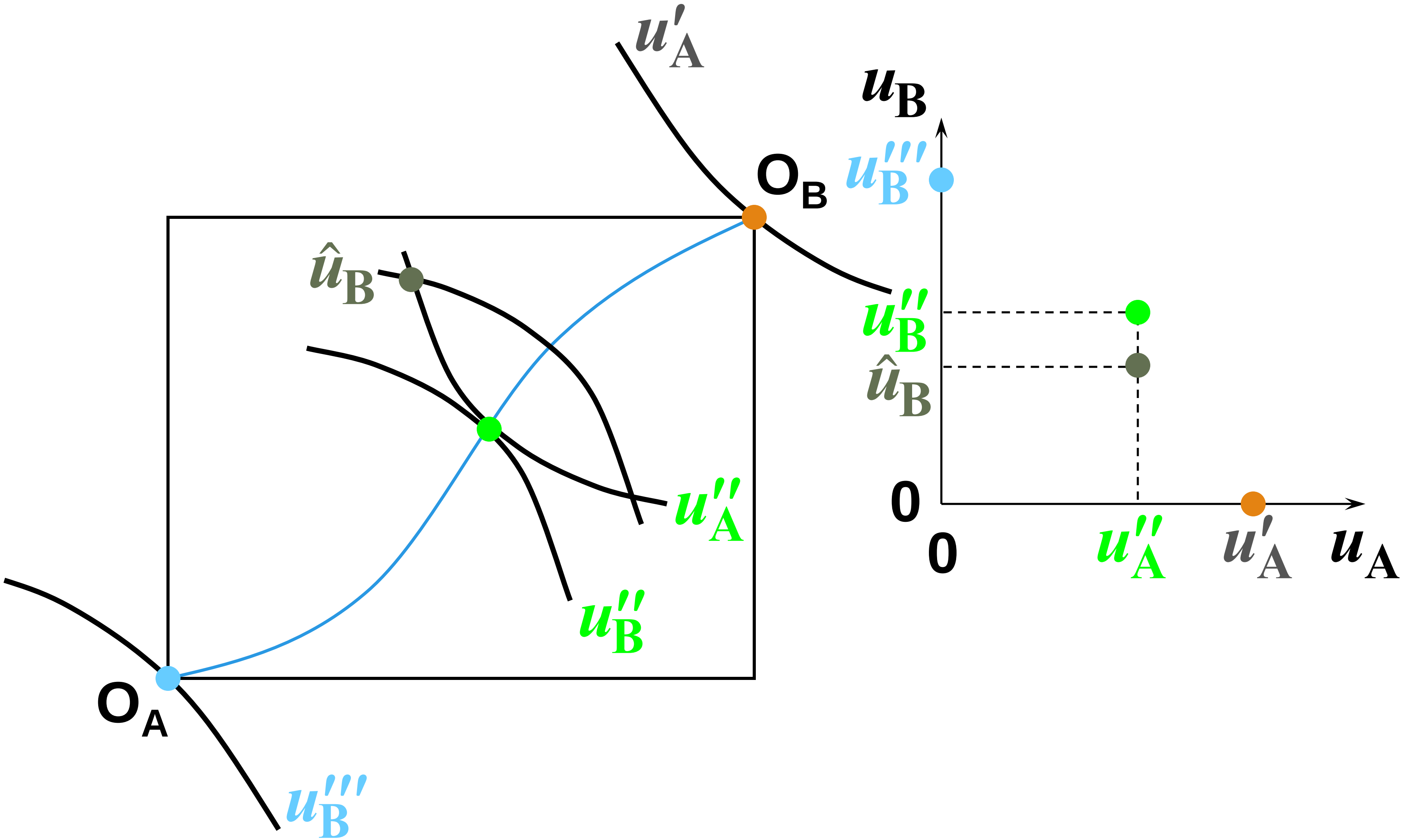

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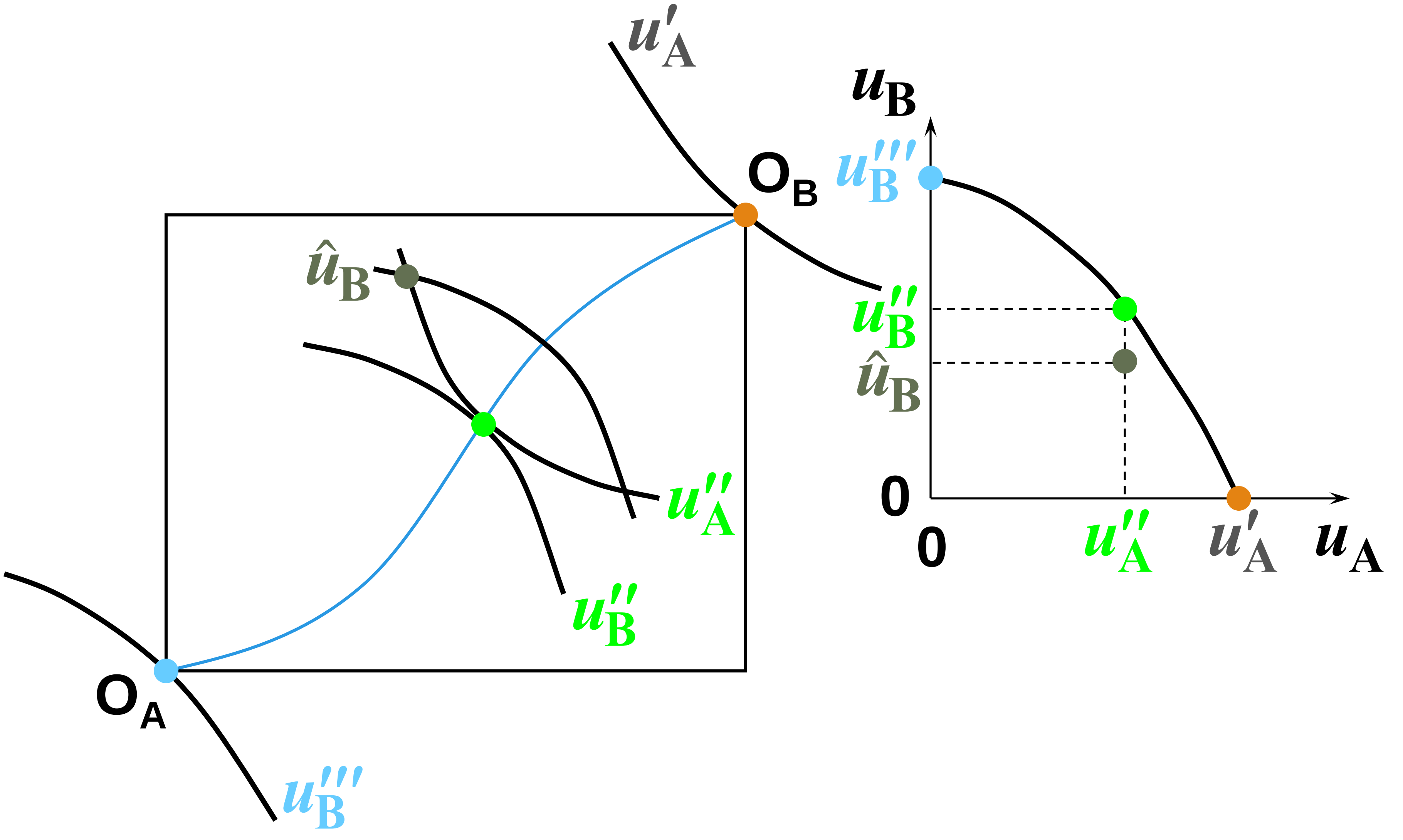

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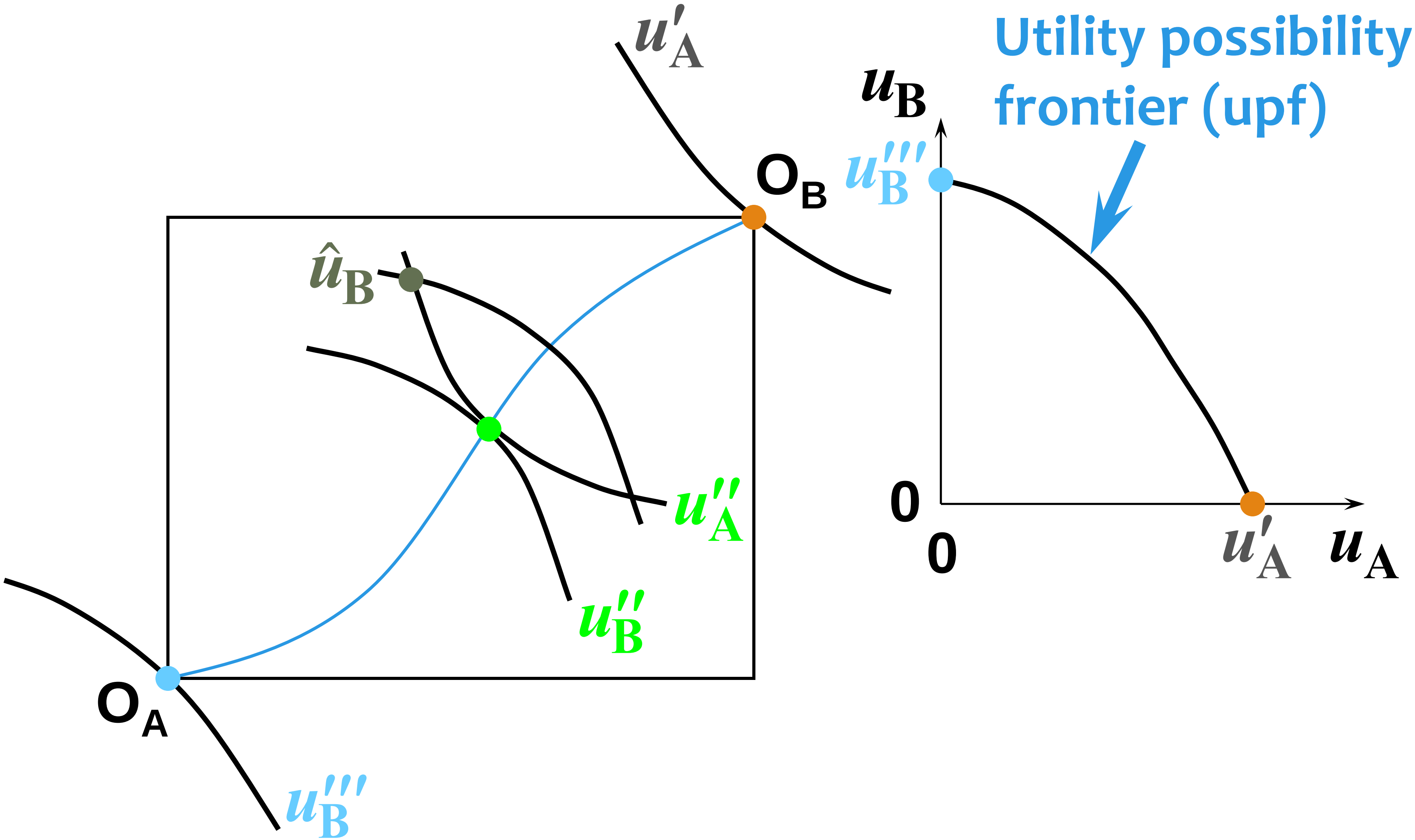

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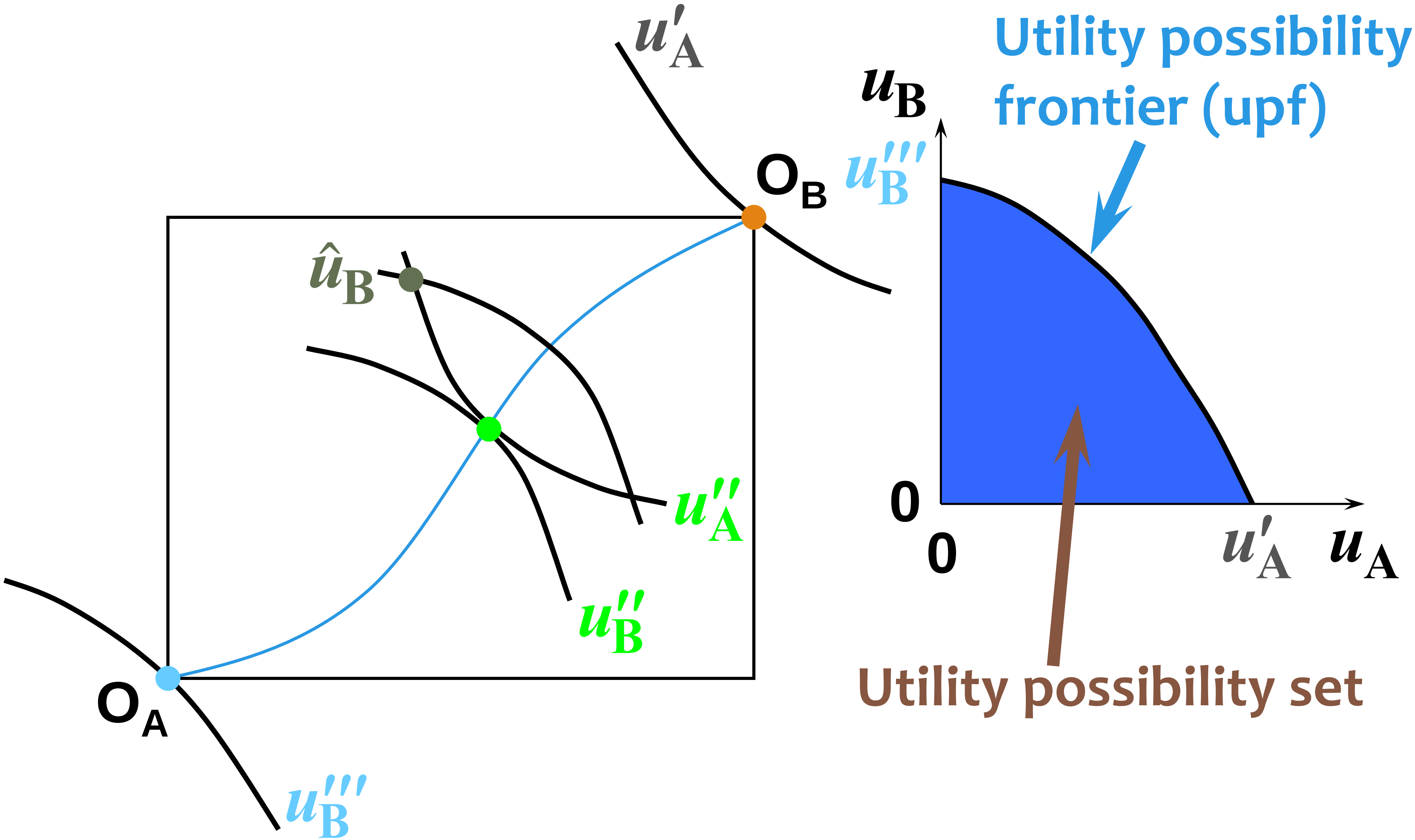
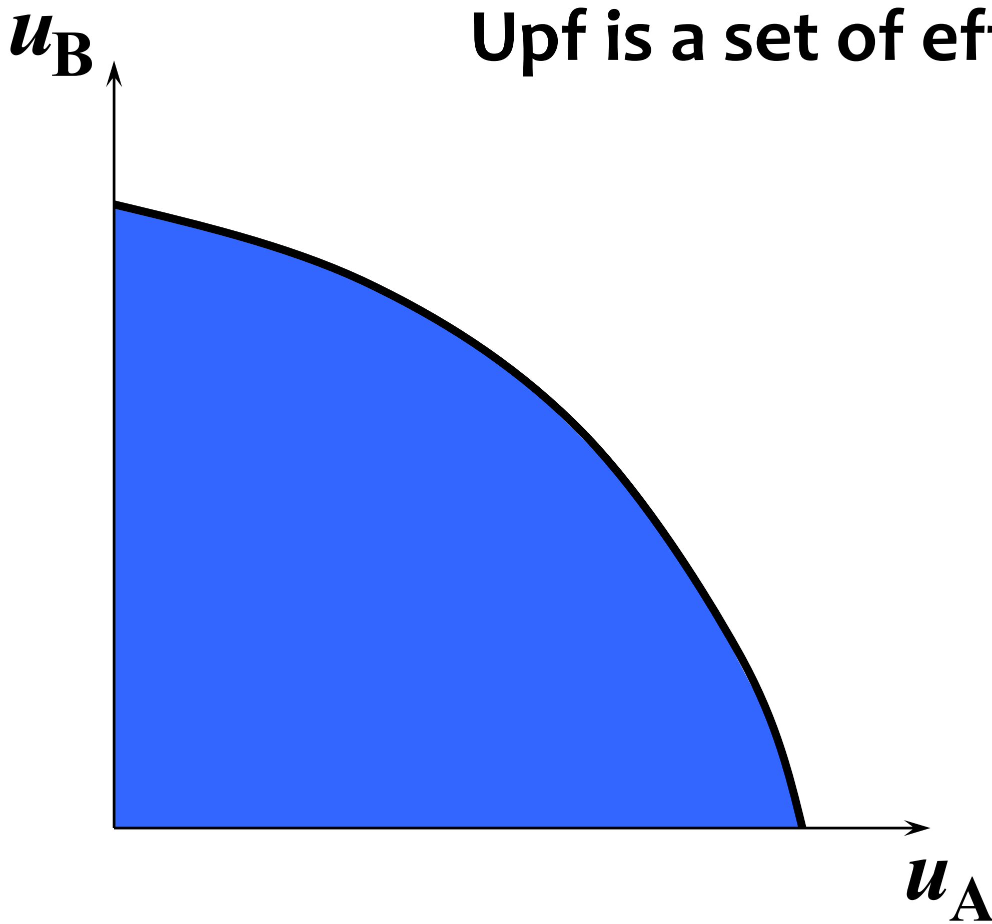


# Social Optima & Efficiency

**Upf is a set of efficient utility pairs.**



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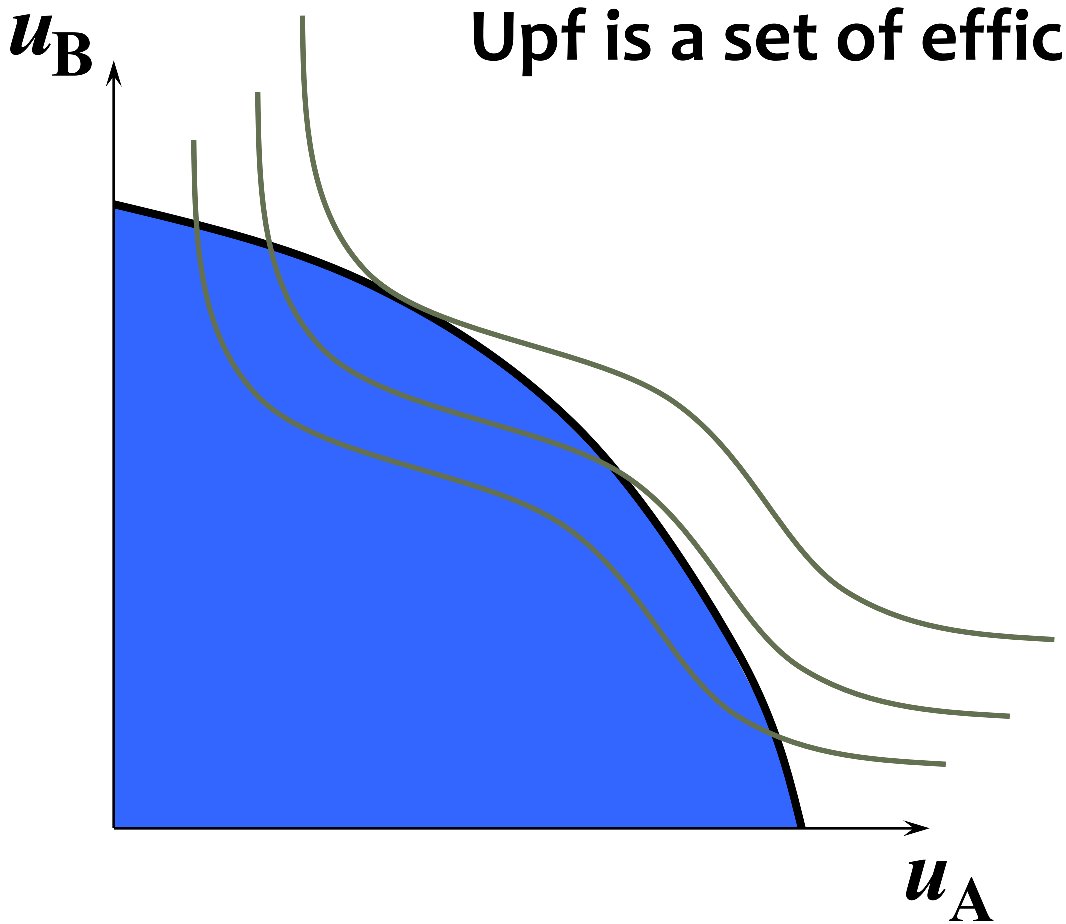


**Upf is a set of efficient utility pairs.**

**Social indifference curves**

(based on a social welfare function; i.e., an aggregated utility function for the whole society)

# Social Optima & Efficiency

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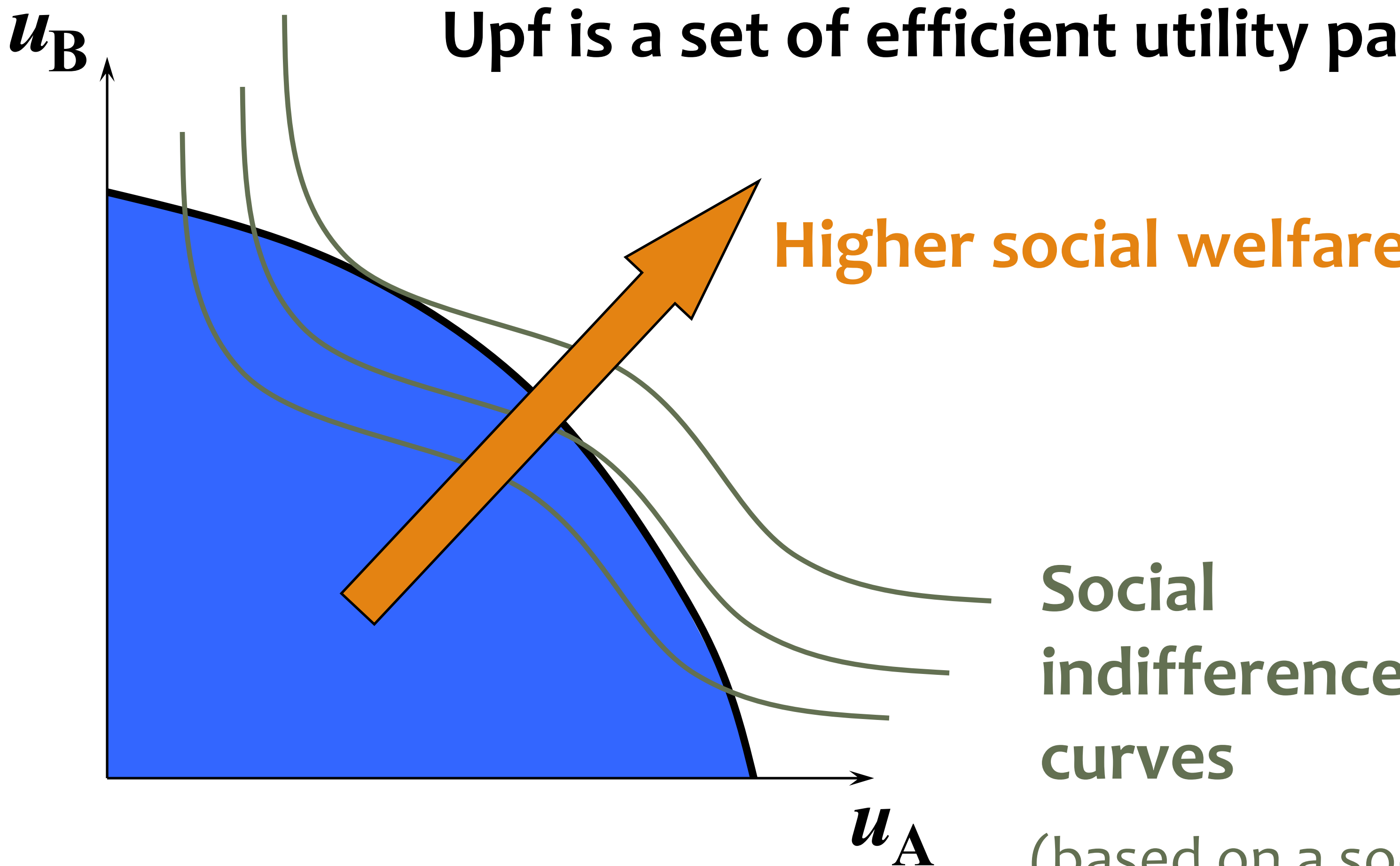


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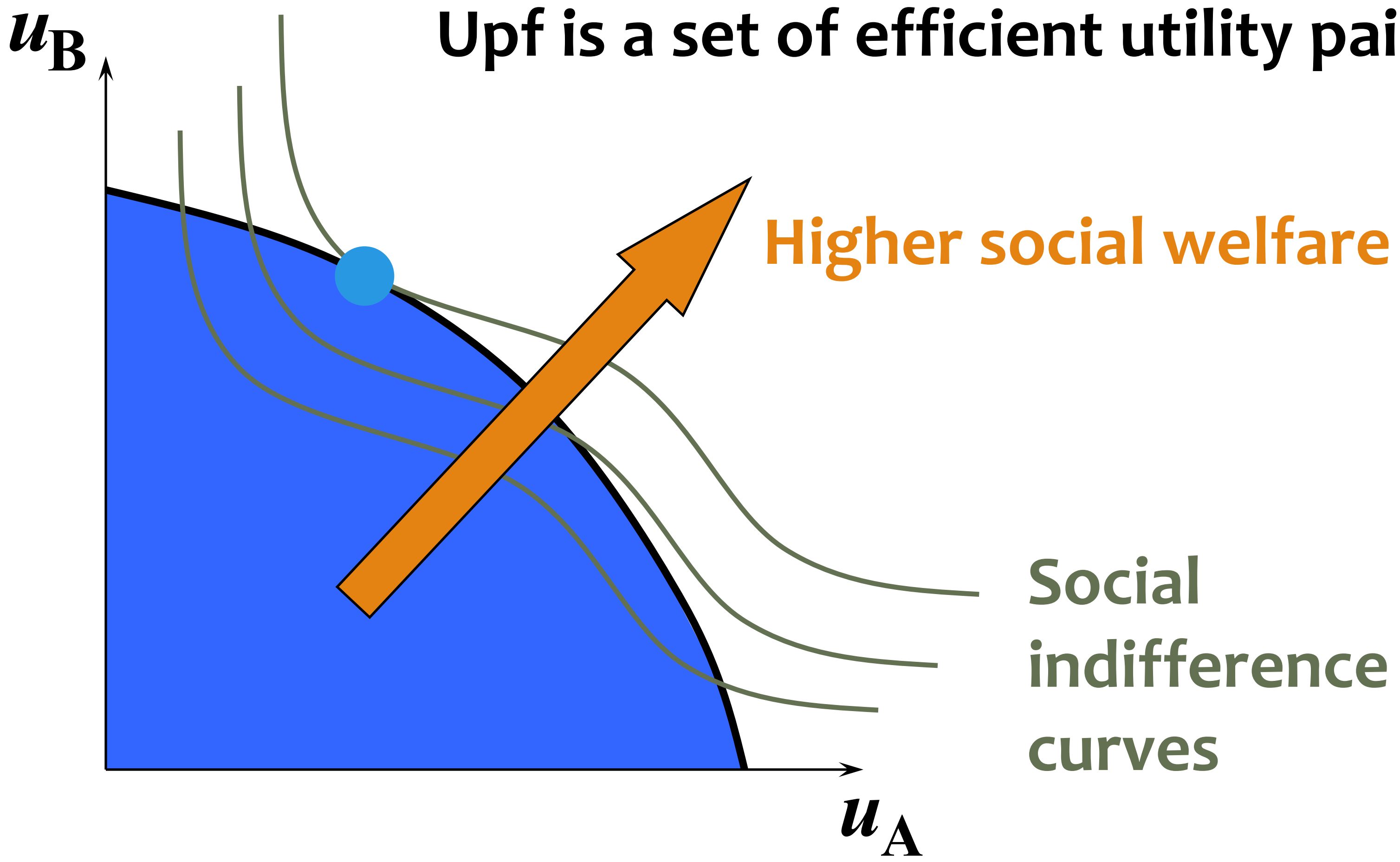

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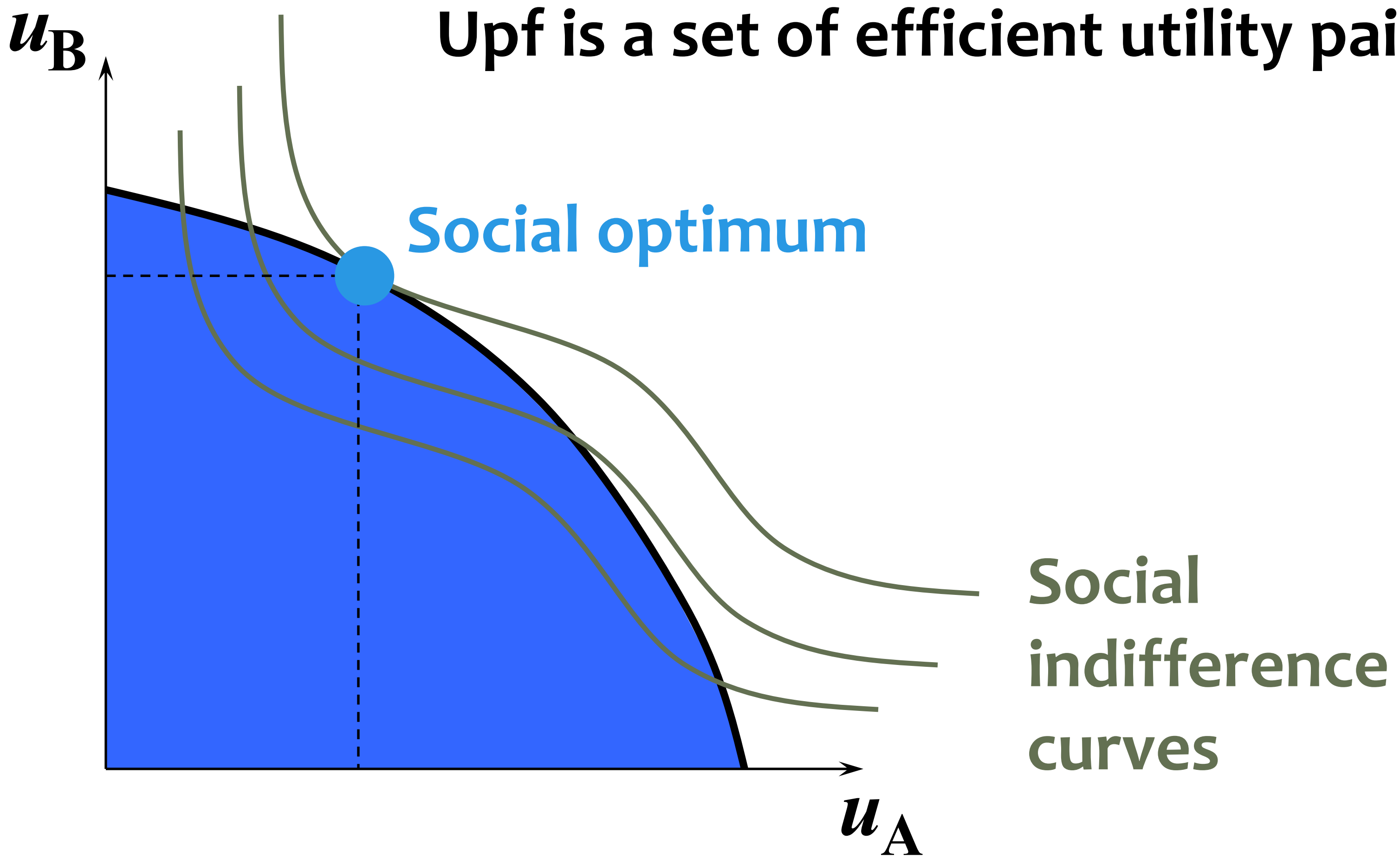

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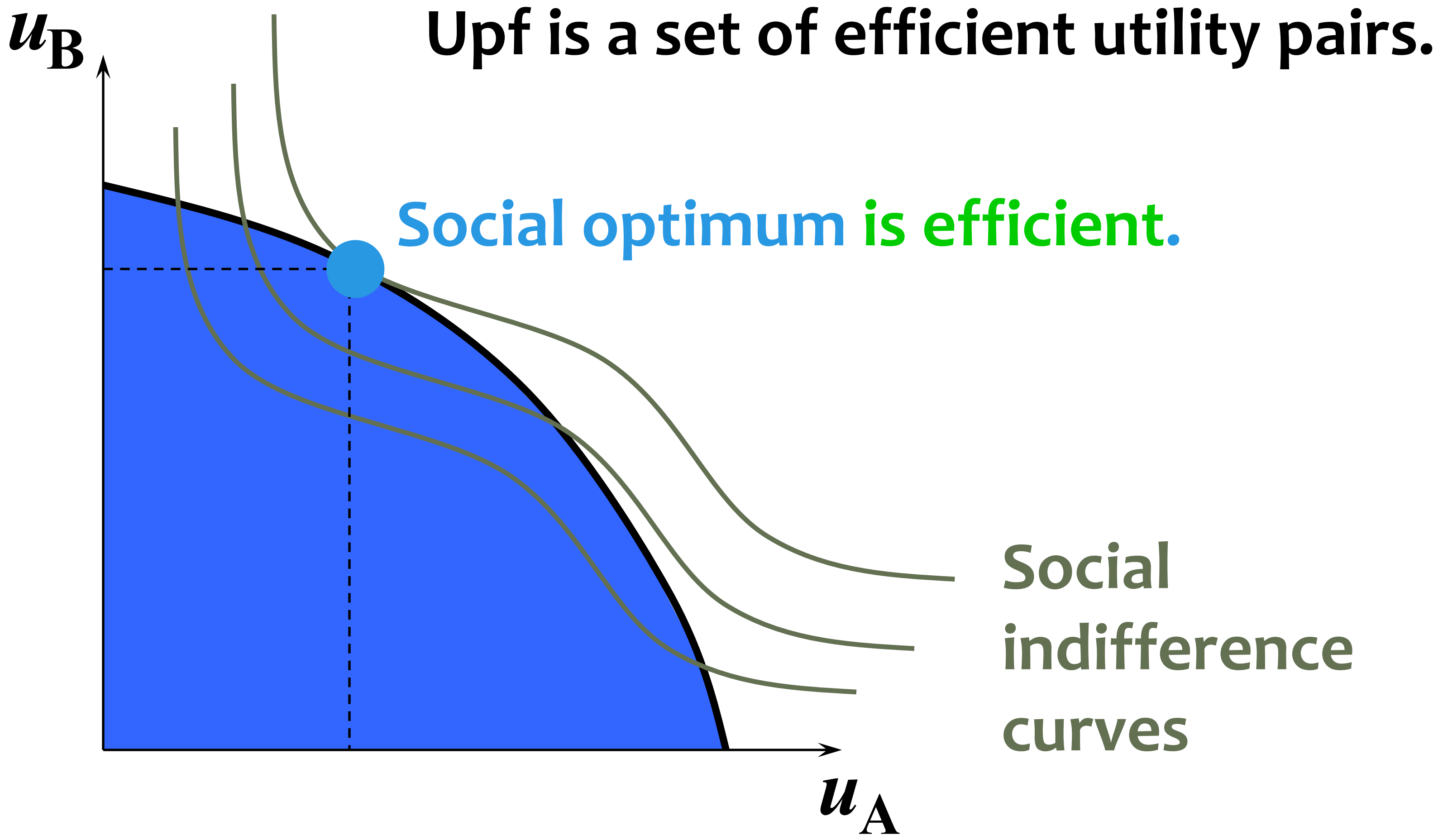

# Fair Allocations

- Some Pareto efficient allocations are “unfair”.
- E.g., one consumer eats everything is efficient, but “unfair”.
- Can competitive markets guarantee that a “fair” allocation can be achieved?
- If agent A prefers agent B’s allocation to his own, then agent A **envies** agent B.
- An allocation is **fair** if it is Pareto efficient and envy-free (**equitable**).

# Fair Allocations

- 2 agents – A and B.
- The same endowments – goods 1 and 2.
- Now trade is conducted in competitive markets.
- Must the post-trade allocation be fair?

# Fair Allocations

- 2 agents – A and B.
- The same endowments – goods 1 and 2.
- Now trade is conducted in competitive markets.
- Must the post-trade allocation be fair?
- Yes. Why?

# Fair Allocations

- Endowment of each agent is $(\omega_1, \omega_2)$.
- Post-trade bundles are $(x_1^A, x_2^A)$ and $(x_1^B, x_2^B)$.
- Then $p_1 x_1^A + p_2 x_2^A = p_1 \omega_1 + p_2 \omega_2$

  and $p_1 x_1^B + p_2 x_2^B = p_1 \omega_1 + p_2 \omega_2$.

# Fair Allocations

- Suppose agent A envies agent B.
- This means that $(x_1^B, x_2^B) \succ_A (x_1^A, x_2^A)$.
- But as $(x_1^A, x_2^A)$ is the best bundle A can afford, this implies $(x_1^B, x_2^B)$ is not affordable for A:

$$p_1 x_1^B + p_2 x_2^B > p_1 \omega_1 + p_2 \omega_2.$$

- Contradiction: Both agents started with equal endowments, so this would mean that agent B cannot afford the bundle $(x_1^B, x_2^B)$ either.

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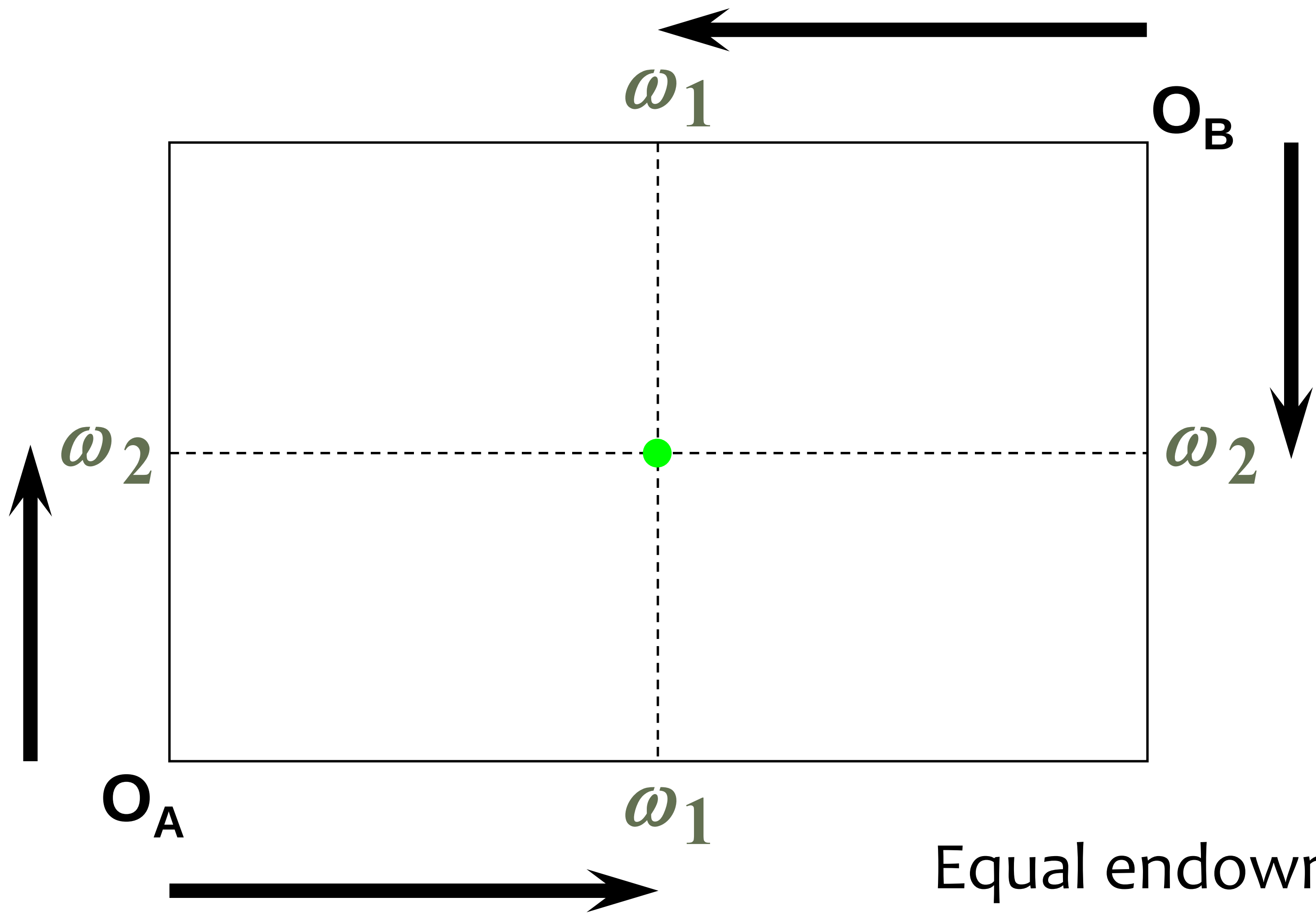
- C… …arted with equal en… …ents, so this would mean that agent B cannot afford the bundle $(x_1^B, x_2^B)$ either.
- Conclusion: It is impossible for agent A to envy agent B in these circumstances.

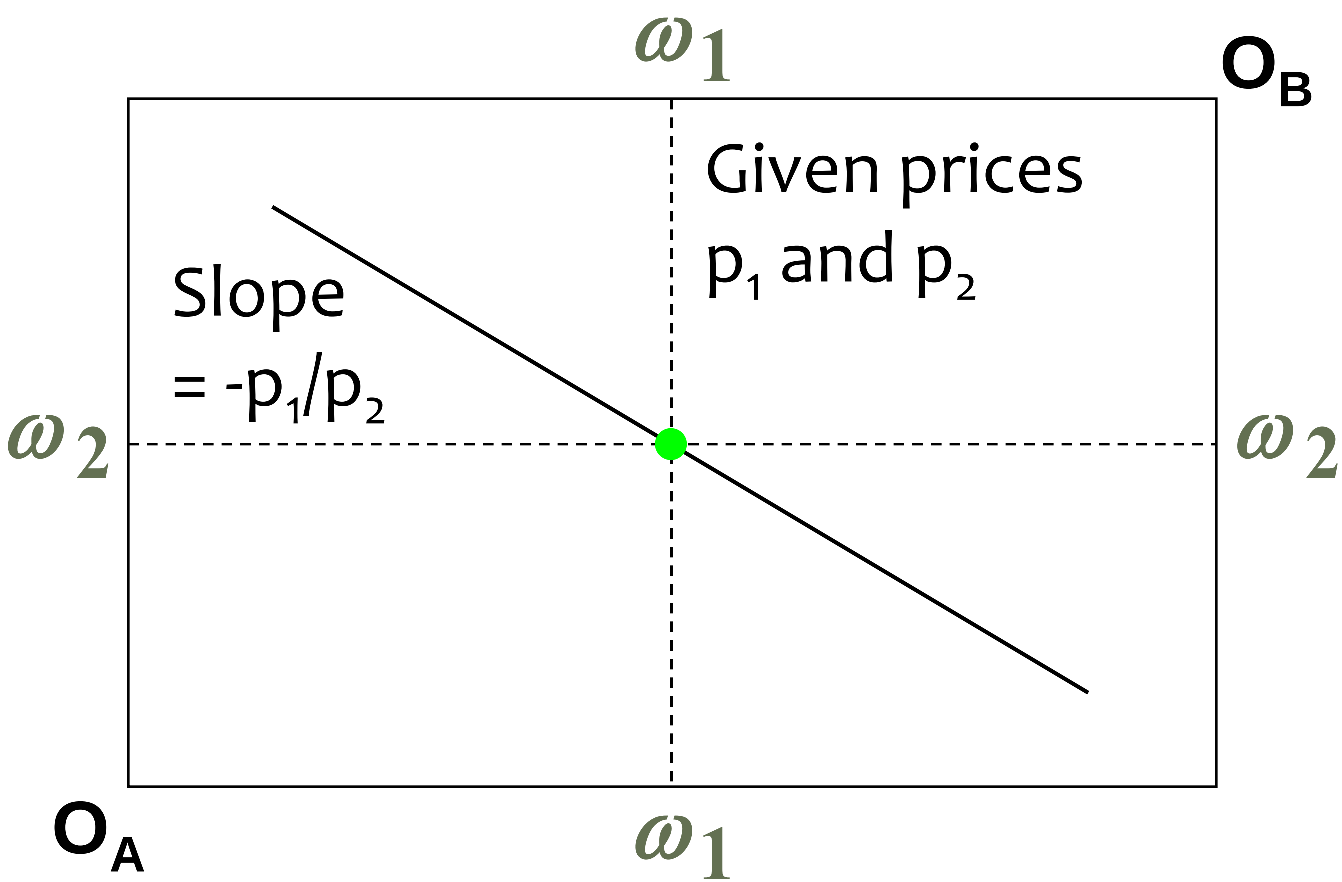
If every agent's endowment is identical, then trading in competitive markets results in a fair allocation.

# Fair Allocations




Equal endowments

# Fair Allocations



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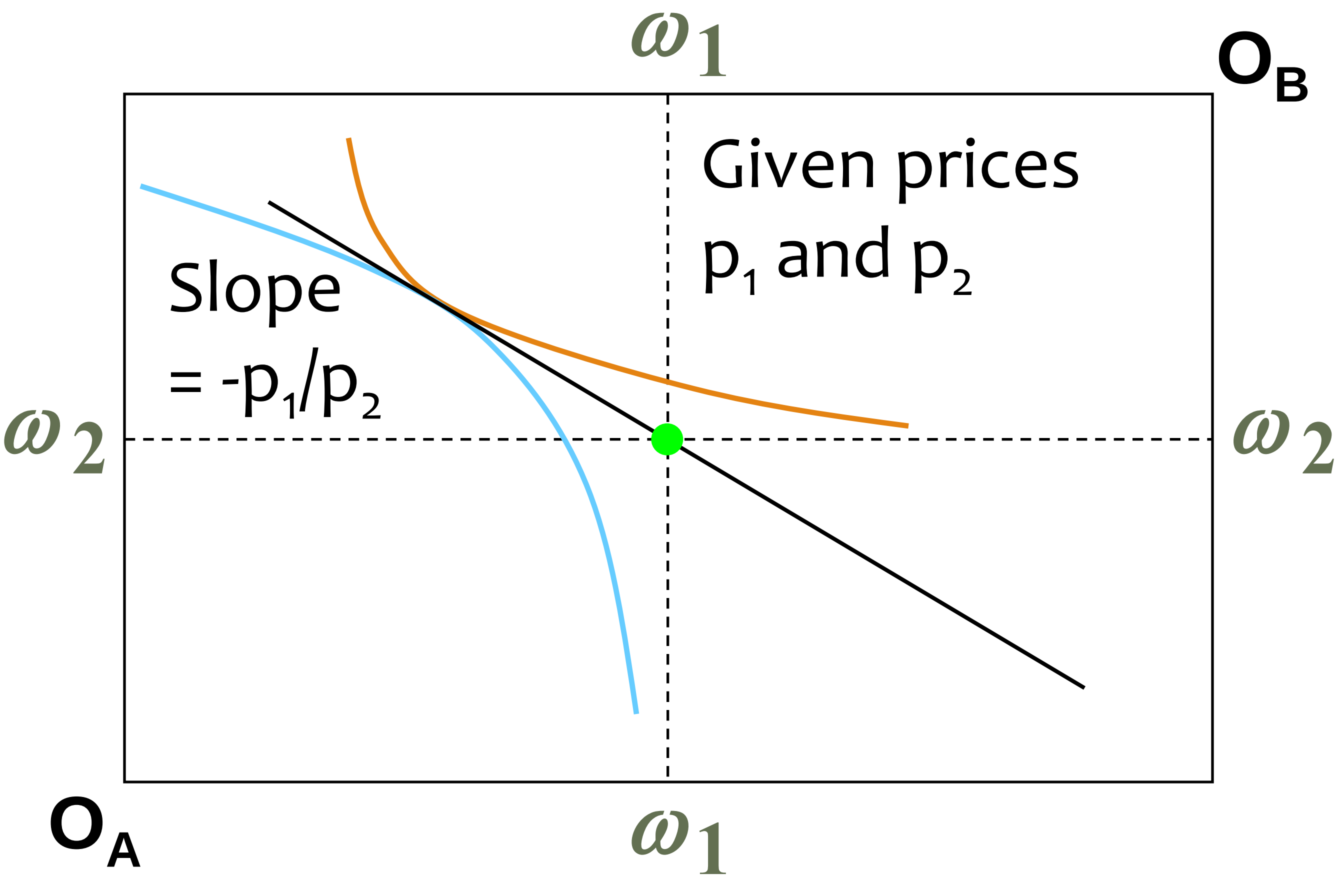

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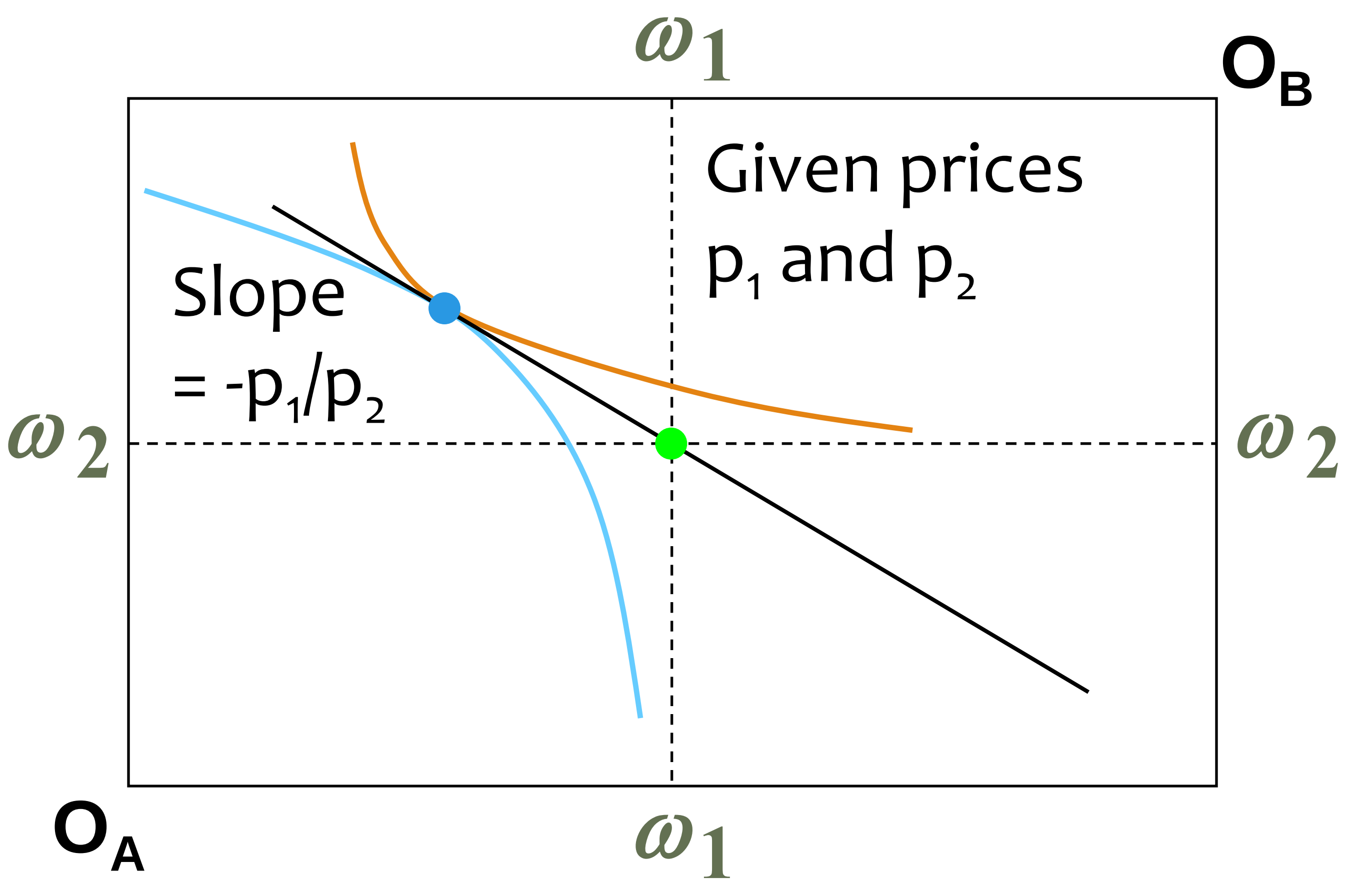

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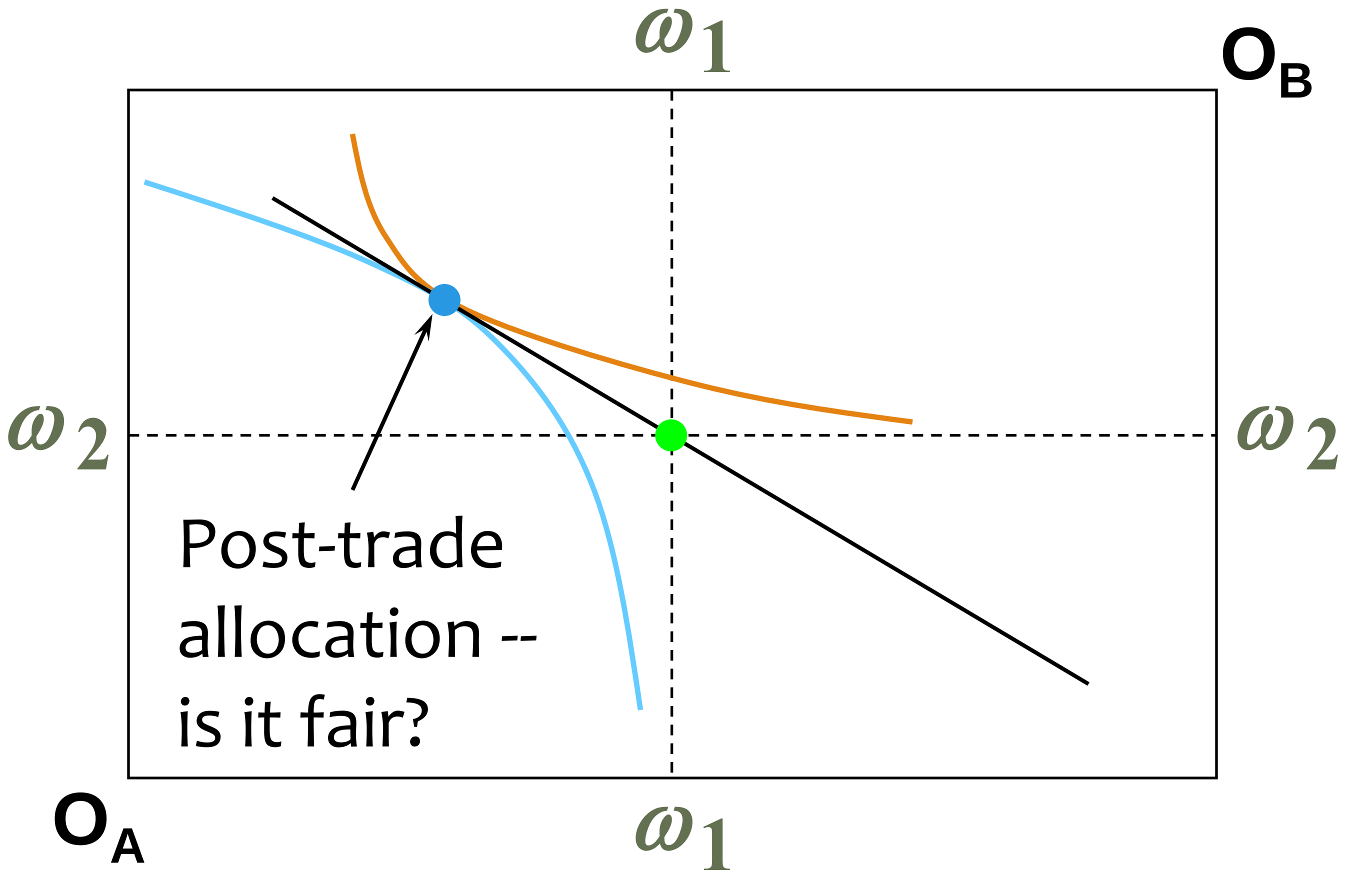

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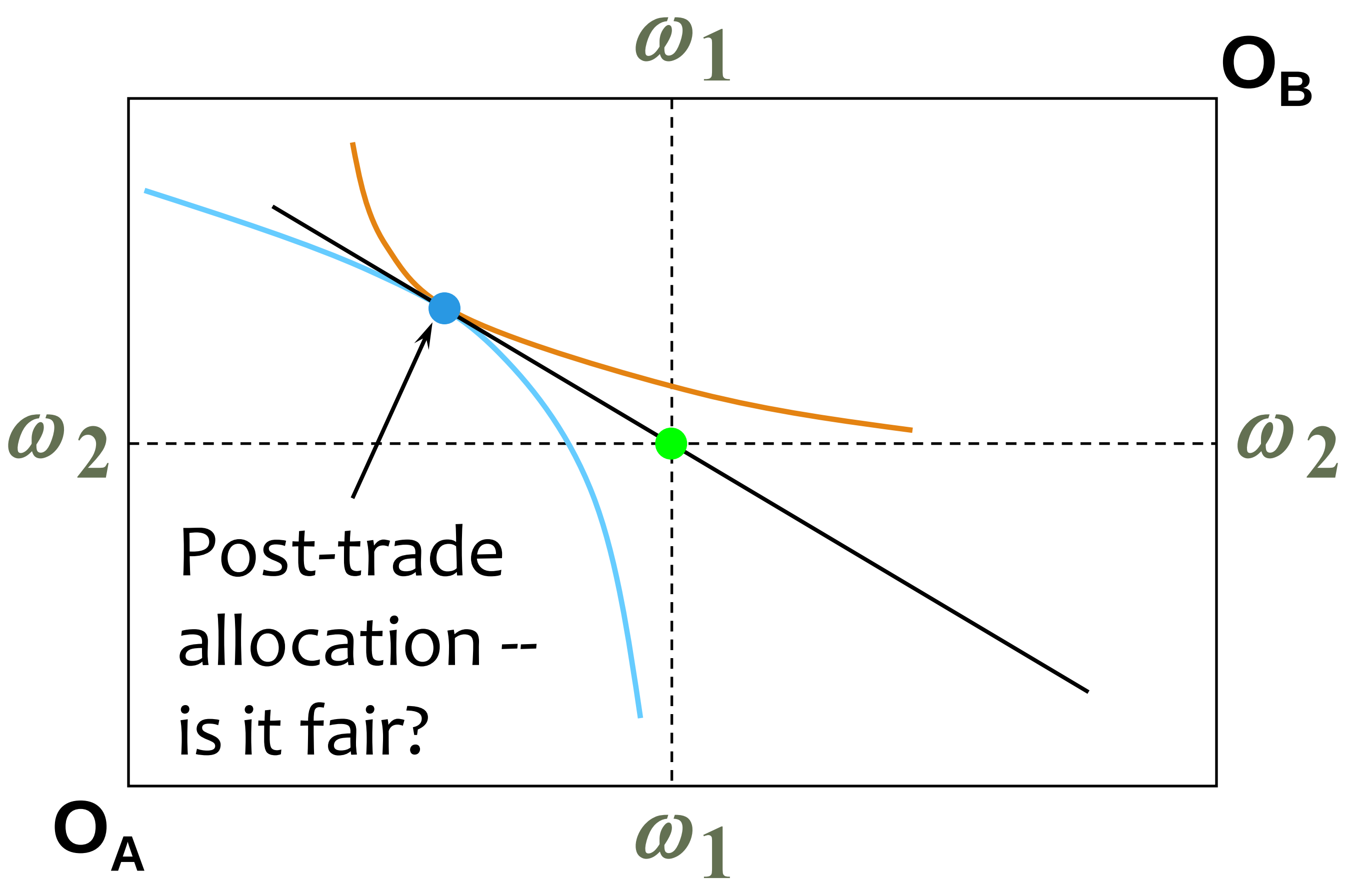
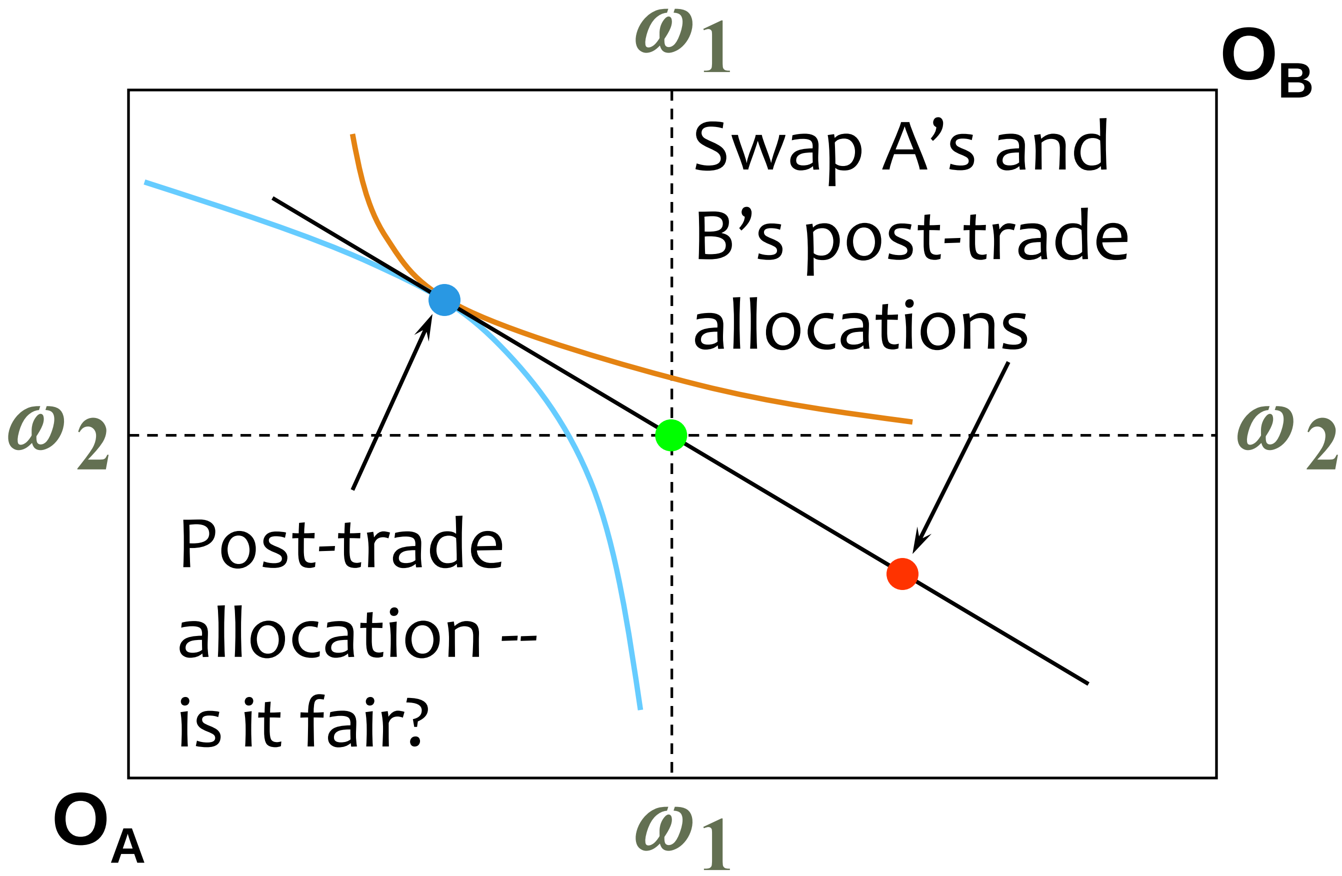



It is Pareto efficient, but does any of the agents envy?

# Fair Allocations

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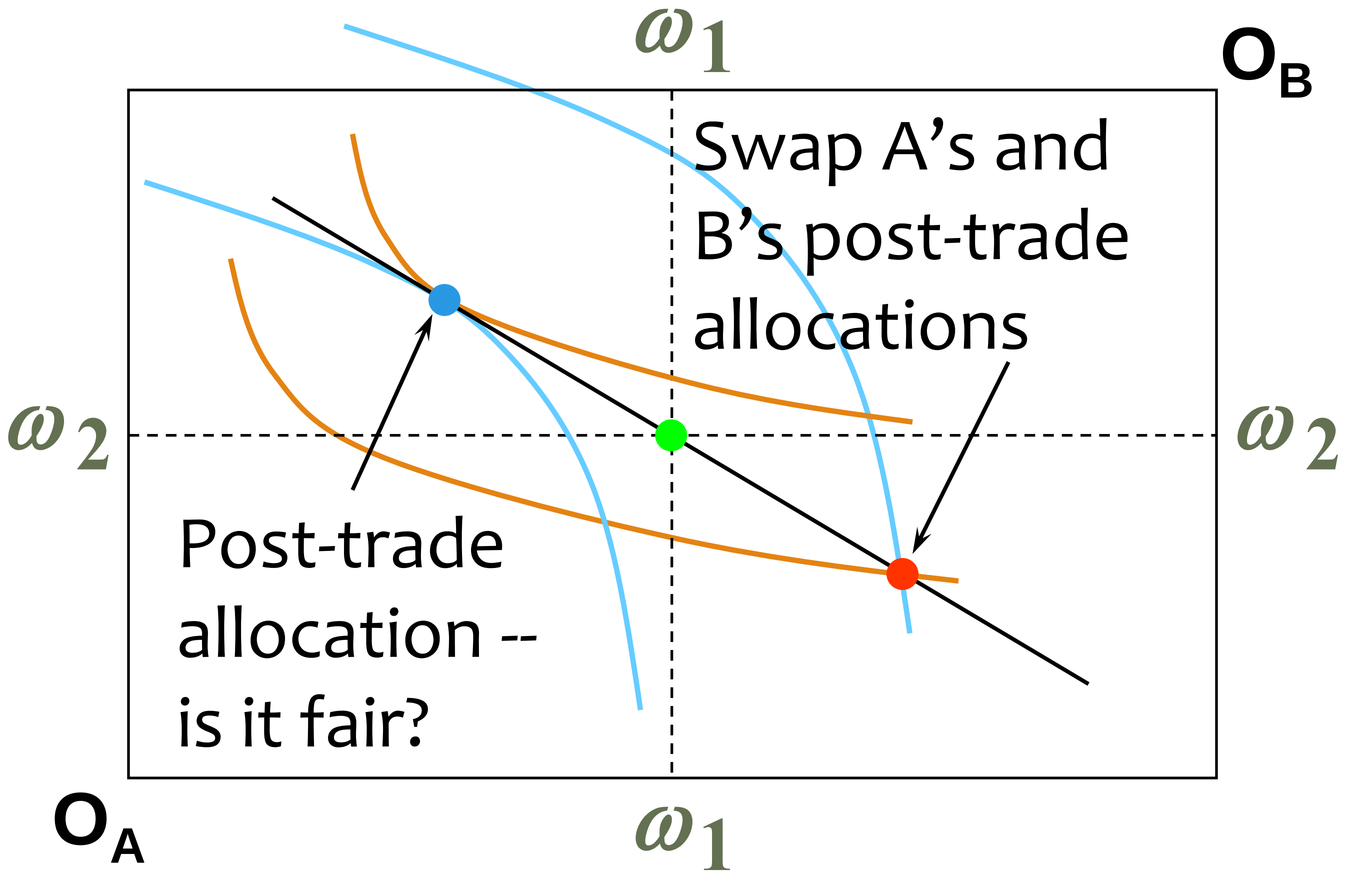
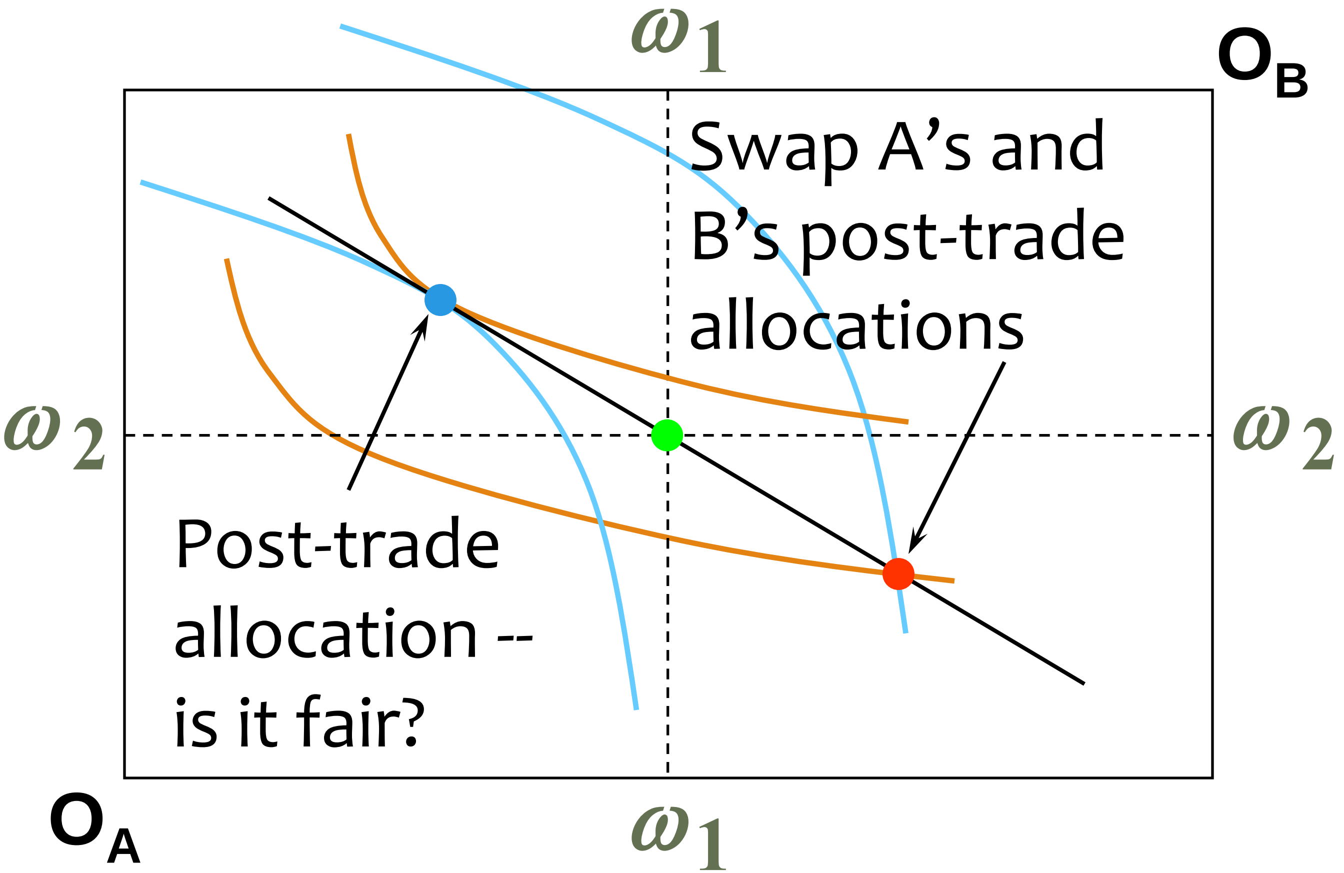



Each person prefers the post-trade allocation to the swapped allocation.

# Fair Allocations




Post-trade allocation is Pareto-efficient and envy-free; hence, it is fair.