Edgeworth box (Leontief)

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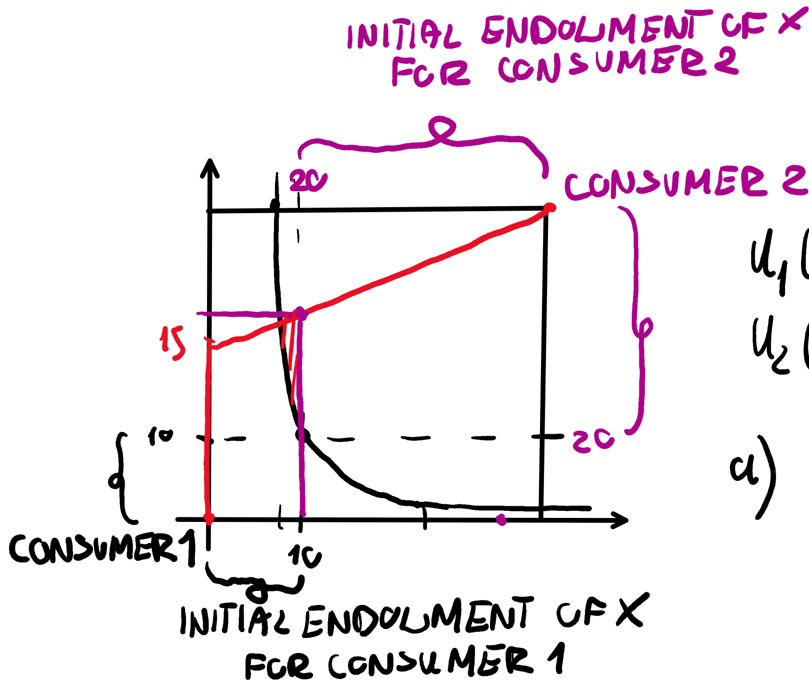

$$U_1(x,y) = x^2 y^{0.5}$$

$$U_2(x,y) = \min(X, 2Y)$$

a)

For Consumer 1 and his initial endowment (10 of X and 10 of Y) his utility is equal to about 316. To draw his indifference curve going through the initial endowment we need to find other bundles of X and Y that give the utility of 316. For example, X = 7.6 and Y = 30, or X = 17.8 and Y = 1. Depicted by black curve.

For Consumer 2 and his initial endowment (20 of X and 20 of Y) his utility is equal to 20. To draw his indifference curve going through the initial endowment we need to find other bundles of X and Y that give the utility of 20. For example, X = 20 and Y = 10, or X = 30 and Y = 10. Depicted by purple curve.

b)

The red area between the curves represents the potential improvement for both consumers

c)

Finding Contract Curve is easy with the Leontief utility function because we know that the following condition needs to hold: $x_2 = 2y_2$

In other words, in the equilibrium, the second consumer will always want to be at the kink point of his indifference curve.

To visualize it may be easier to express it in terms of x1 and y1. We can use the following set of equations:

$$\begin{cases} x_2 = 2y_2 \\ x_1 + x_2 = 30 \\ y_1 + y_2 = 30 \end{cases} \Rightarrow (30 - x_1) = 2(30 - y_1) \Rightarrow$$

$$\Rightarrow \boxed{y_1 = 15 + \frac{x_1}{2}}$$

This is depicted with a red line in a figure above.

d) AT EQUILIBRIUM:

$$\begin{cases} MRS_1 = \frac{P_x}{P_y} \\ P_x x_1 + P_y y_1 = 10(P_x + P_y) \end{cases} \Rightarrow \begin{cases} 4\frac{y_1}{x_1} = \frac{P_x}{P_y} \\ x_1 = \frac{10(P_x + P_y) - P_y y_1}{P_x} \end{cases} \Rightarrow$$

$$\Rightarrow y_1 = \frac{P_x}{4P_y} \cdot \left(\frac{10(P_x + P_y) - P_y y_1}{P_x}\right) \Rightarrow \begin{cases} y_1 = 2\left(\frac{P_x}{P_y} + 1\right) \\ x_1 = 8\left(\frac{P_y}{P_x} + 1\right) \end{cases}$$

We can then find the price ratio by substituting these demands into a contract curve:

$$2\left(\frac{P_x}{P_y} + 1\right) = 15 + 4\left(\frac{P_y}{P_x} + 1\right) \Rightarrow \frac{P_x}{P_y} \approx 8.73 \Rightarrow$$

$$\Rightarrow \begin{cases} y_1 \approx 19.46 \\ x_1 \approx 8.92 \end{cases} \text{AND SO:} \begin{cases} y_2 = 10.54 \\ x_2 = 21.08 \end{cases}$$