**SOCIAL WELFARE**

**Problem 1**

A community consists of two types individuals (*A* and *B*), and the utility possibility frontier is given by $U_A + 2U_B = 200$. Plot the utility frontier. For each point below, determine values of $U_A$ and $U_B$ that maximize the social welfare ($W$). Show the solutions both algebraically and graphically.

a) Assume a "Nietzschean social welfare function": $W(U_A; U_B) = \max\{U_A; U_B\}$.

b) Use a Rawlsian criterion: $W(U_A; U_B) = \min\{U_A; U_B\}$.

c) Suppose social welfare is given by $W(U_A; U_B) = U_A^{1/2} U_B^{1/2}$.

**Problem 2**

There is a group of parents that have two children, named A and B, and they love both of them equally (this means, that each PLN spent on child A has the same value as one PLN spent on child B, or in other words, that each child has the same functional form as the other one-see the utility functions). Each of the parents in this group receives a benefit of 1,000 PLN to distribute between the children (or on their education, etc). Each parent is a different person, thus has a different moral code that guides their behavior (though all of them want the best possible for their children, they just may have a different belief in what that means). Thus, they have different utility functions. Based on the below utility functions, determine how will they divide the money between the children in each case.

a) $U(a; b) = \log(a) + \log(b)$

b) $U(a; b) = \min\{a,b\}$

c) $U(a;b)=a^2 +b^2$

d) $U(a; b) = - 1/a - 1/b$

e) $U(a; b) = \max\{a,b\}$

f) $U(a; b) = a^{0.5} + b^{0.5}$

**Problem 3**

Robert and John each Friday go to this one Italian restaurant, where their favorite dish is spaghetti. Robert likes spaghetti, but he also likes John to be happy and he knows that spaghetti makes him happy. John likes spaghetti, but he also likes Robert to be happy and he knows that spaghetti makes Robert happy. Robert's utility function is $U_R(S_R; S_J) = S_R^{\alpha} S_J^{1-\alpha}$ and John's utility function is $U_J (S_J; S_R) = S_J^{\alpha} S_R^{1-\alpha}$, where $S_J$ and $S_R$ are the amount of spaghetti for Robert and the amount of spaghetti for John, respectively. There is a total of 24 units of spaghetti to be divided between them (think of this as one big plate of spaghetti, which they then divide between each other).

a) Suppose that $\alpha = 2/3$. If Robert got to allocate the 24 units of spaghetti exactly as he wanted to, how much would he give himself, and how much would he give John?

b) Again assuming $\alpha = 2/3$, if John got to allocate the spaghetti exactly as he wanted to, how much would he take for himself, and how much would he give Robert?

c) What are the Pareto optimal allocations when $\alpha = 2/3$? (Hint: An allocation will not be Pareto optimal if both persons' utility will be increased by a gift from one to the other.)

d) Suppose that $\alpha = 1/3$. If Robert got to allocate the spaghetti, how much would he give himself? If John got to allocate the spaghetti, how much would he give himself?

e) When $\alpha = 1/3$, at the Pareto optimal allocations what do Robert and John disagree about?

**Problem 4**

One possible method of determining a social preference relation is rank-order voting (also known as the *Borda count*). Each voter is asked to rank all of the alternatives, and the voters' scores for each alternative are then added over all voters. The total score for an alternative is called its Borda count. For any two alternatives, *x* and *y*, if the Borda count of *x* is smaller than the Borda count for *y*, then *x* is socially preferred to *y*.

Suppose that there are a finite number of alternatives to choose from and that every individual has complete, reflexive, and transitive preferences. For the time being, let us also suppose that individuals are never indifferent between any two different alternatives but always prefer one to the other.

a) Is the social preference ordering defined in this way complete, reflexive, and transitive?

b) If everyone prefers *x* to *y*, will the Borda count rank *x* as socially preferred to *y*? Explain your answer.

c) Suppose that there are two voters (1 and 2) and three candidates (*x*, *y*, and *z*). Suppose that Voter 1 ranks the candidates, *x* first, *z* second, and *y* third. Suppose that Voter 2 ranks the candidates, *y* first, *x* second, and *z* third. What is the Borda count for each of the three candidates?

d) Some new information is discovered about the candidate *z*. Voter 1 is appalled by this information and changes his ranking to *x* first, *y* second, *z* third. Voter 2 is favorably impressed and changes his ranking to *y* first, *z* second, *x* third. Now what is the Borda count for each of the three candidates?

e) Does the social preference relation defined by the Borda count have the property that social preferences between *x* and *y* depend only on how people rank *x* versus *y* and not on how they rank other alternatives? Explain (you can use the examples from points c and d).

**Problem 5**

Two parts of a town hate each other due to historic reasons. However, to provide energy for the town they need to share between each other the available energy generated by the local plant, and each part of the town wants the other one to have as little as possible (the less other part has, the more it increases their own utility). North's utility function is $U_N(W_N;W_S) = W_N - W_S^2$ and South's utility function is $U_S(W_S;W_N) = W_S - W_N^2$, where $W_S$ is South's energy consumption and $W_N$ is North's energy consumption. The plant generates 4GW of energy.

a) If South got to allocate all of the energy, how would they allocate it?

b) If North got to allocate all of the energy, how would they allocate it?

c) If each of them gets 2GW of the energy, what will the utility of each of them be?

   If there was a major disruption in the plant, and it was only able to generate 2GW of the energy and they divided the remaining 2 GW equally between them, what would the utility of each of them be?

d) If it is possible to "throw away" some energy and they must consume equal amounts of energy, how much should they throw away?